ROUTE SURVEYING

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THIRD EDITION

New York · JOHN WILEY & SONS, Inc.

London · CHAPMAN & HALL, Limited

PRINTED IN THE UNITED STATES OF AMERICA

PREFACE

The untimely death of Professor Pickels on December 2, 1944, interrupted a program of revisions intended to bring this book up to date and to make it more usable for both students and field engineers. The work, however, has been continued, and the results are now presented in this new volume.

The general character and makeup of the book remain the same. Revisions of greater or lesser extent, however, appear in many places. They are designed to clarify certain points, to present new concepts and methods, and to give new data. All of these increase the value of the volume, either as textbook or reference book.

The largest single revision is Chapter 6 on String-lining, which has been completely rewritten. The object has been to present more fully and clearly the basic principles involved and the reasons behind each operation rather than merely to give the mechanical procedures to be followed. In addition, both the Portser and the A.R.E.A. methods have been included instead of only the latter as in the Second Edition. It is believed that this new presentation will enable the beginner to gain a clear conception of the theory and processes of string-lining from which he may develop skill and proficiency by practice and experience.

In Chapter 3 on Distance, Curvature, and Grades the latest data on freight-train resistance are given in graphical form, together with charts relating to locomotive resistance, tractive effort, and horsepower.

A short revision in Chapter 4 on Circular Curves presents the basic conception of the degree of curve as the unit of curvature of the circle, to which the usual "angle at the center" is a corollary, and indicates how the degree of curve may be applied to other curves, such as the spiral. The chapter also contains a section on the adjustment of field errors.

Chapter 5 on Spirals contains a more clearly presented and simplified procedure for computing the deflection angles when a set-up is made on the spiral and also a very simple and direct method for determining the orientation angle for such a set-up when the backsight must be taken on some point on the spiral other than the *T.S.* This process again emphasizes the simplicity of the spiral when attacked from the standpoints of speed and curvature rather than length and radius of curvature.

In Chapter 9 on Earthwork the method of determining the areas of cross-sections and the location of slope stakes by plotting the sections and using the planimeter are given in detail.

A number of important changes have been made in the Tables. First, all logarithmic tables have been omitted. Calculating machines have largely displaced manual computations, and a canvass of engineers, teachers, and students indicates that, even before the advent of the calculators, logarithmic solutions were rarely used in route surveying. Consequently the conclusion was reached that logarithmic tables in a book of this kind are not worth the space they occupy.

With the increasing use of flatter curves on both railroads and highways and the demands for greater accuracy it has been found that many computations can not be made with the requisite precision by the use of five-place tables. Therefore, the five-place tables of the natural trigonometric functions have been replaced with seven-place tables, including the addition of a table of secants and cosecants.

Table 8, Cubic Yards per 100 Feet, replacing old Table 19, has been extended to cover side slopes and widths of roadbed more adaptable to present requirements of both railroads and highways. Table 9 is old Table 20.

Table 14 is a new table giving minutes in decimals of a degree and, what is numerically the same, minutes of time in decimals of an hour. This table is often useful in conjunction with Table 13, Functions of a 1° Curve, and also in problems relating to speeds and schedules.

Acknowledgment is hereby made of the courtesy of Mrs. Howard C. Ives in permitting the use of Tables 15, 16, 17, and 18 from her late husband's book "Highway Curves"; of my colleague, Professor W. W. Hay in supplying much valuable advice and assistance in rewriting Chapter 6; and of the Illinois Central Railroad and the Electromotive Division of the General Motors Corporation in furnishing data on locomotive tractive effort and horsepower.

This new volume is affectionately dedicated to the memory of my former friend, colleague, and co-author, the late Professor George Wellington Pickels, Jr.

Carroll C. Wiley

University of Illinois,
Urbana, Illinois,
January, 1949.

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ROUTE SURVEYS

1. Introduction. Route surveying is that branch of general surveying which treats of the field work and calculations required in the location, construction, and maintenance of routes of transportation and communication such as railroads, highways, canals, pipe lines, and electric transmission lines.

Although such routes may extend over large distances, the width of way in every case is very narrow, and hence the shape of the earth affects only the absolute direction, not the relative direction, of the lines which compose the route. Consequently, the methods of plane surveying are entirely adequate for route surveys and are universally used.

2. Classification of Terrain. The practices in route surveying differ greatly in different parts of a country due partly to the nature of the particular project, partly to local preference, but principally to differences in topography.

All country can be divided broadly into three classes:

1. Level, prairie country which offers few or no obstacles in the way of hills, valleys, etc., and consequently allows the locating engineer much latitude in the placing of the line to meet the established needs.
2. Rolling, hilly country through which several lines are possible, none of them departing to any great extent from the direct line between controlling points, but which require careful investigation by the locating engineer to choose the best one.
3. Mountainous country where the judgment, skill, and ingenuity of the locating engineer are taxed to find a satisfactory route, or in some cases even a possible route.

Obviously there is no sharp division between these various classes

of country, and one class may merge into another almost imperceptibly. Likewise any or all of them may be found on any given project, and hence such a project may call for familiarity with the methods applicable to each.

3. Kinds of Survey. The different surveys may be classified by the character of the route to be established, as Railroad Surveys, Highway Surveys, Pipe-line Surveys, etc. These surveys, however, do not differ greatly in their general characteristics. Such differences as do occur are due principally to the limitations of alinement and grade permissible for each kind of work, and to a less degree to the topographic features, especially existing improvements, or established routes of similar character. In general, however, each of these basic projects calls for a sequence of specific surveys as follows:

1. A reconnoissance survey, or a rapid general survey for the purpose of selecting practicable routes.
2. A preliminary survey, with the usual surveying instruments, for selecting the particular route and for designating the alinement, grades, and structures.
3. A location survey for establishing the alinement on the ground.
4. Construction surveys for reestablishing the alinement where necessary and for laying out the various details of the project.
5. Maintenance surveys for the upkeep and improvement of the project after it is once built.

Railroad location and construction show these various surveys in their most complete and comprehensive form, hence railroad surveys will be treated first, and in detail, and the surveys for other kinds of routes will be given by suggesting the application or modification of the methods of railroads surveys to these other kinds of surveys.

RAILROAD SURVEYS

Railroad Reconnoissance Surveys

The terminals and intermediate points connected by a railroad are determined by its promoters with a view to the amount of traffic that can be expected from them. The nature and the amount of the traffic and the direction of heaviest haul determine

the maximum gradients and curvature advisable. The first thing to be determined by the locating engineer is the directions of the lines joining controlling points. This information can be obtained with considerable accuracy from the United States Geological Survey maps of the territory, or similar maps if such are available. Otherwise, such maps as can be obtained locally will have to be used.

4. Reconnoissance. A reconnoissance is then made of the strip of country through which the road is to pass, as the result of which some of the routes are eliminated as impracticable and one or more are chosen for a more detailed survey.

If the country is of the first class, a reconnoissance is rarely necessary, and the first survey is in the nature of a preliminary.

For country of the second class, the reconnoissance should be made across-country, following, as closely as possible, the direction of the line determined from the map. Usually the trip is made on foot or on horseback, but occasionally an automobile may be used. If in following the direct line between controlling points, obstacles are met with which can not be surmounted, such as high hills, the engineer should explore on both sides of the obstacle and decide, if possible, which route offers the least resistance. Before deflecting from a straight line, the engineer must be sure that his reasons for doing so are justified from an economic standpoint. In country of the second class it is seldom that grades cause much trouble, and the main lookout of the engineer is to keep the amount of curvature as low as possible. The result of the reconnoissance through country of the second class is that one or more routes are selected for a more detailed survey.

It is in mountainous country—third class—that all the skill of the engineer is brought into play. The drainage of the country should be carefully studied, as it plays a very important part in the location of a railroad. If the controlling points are in the same valley, the main problem is solved, and the conforming of the alinement to the topography is merely a matter of detail. But when the controlling points are in different valleys, the ridges between them have to be crossed, and the principal object of the reconnoissance is to discover the most favorable crossing places in the valleys and on the ridges. The saddles in the ridges and the most favorable river crossings become secondary controlling points. The locations of all such points are platted on the map, and their elevations and distances apart are recorded. The most important and useful

"instruments" used on reconnoissance are the judgment and experience of the locating engineer, as upon these depend the extent and the cost of more detailed surveys and the cost of construction and of operation.

Recently aerial survey methods have been adapted to railroad reconnoissance and preliminary surveys. Observation flights, aided by aerial photographs of the more important portions of the line, serve many, if not all, of the needs of reconnoissance, and can be made with the minimum expenditure of time and effort, and usually at a smaller cost.

Railroad Preliminary Surveys

5. Preliminary. In country of the first class, a preliminary line is run for direction. The magnetic bearing of the direct line between controlling points is measured on the map, and a line having this direction is initiated from the first controlling point and produced to the second controlling point. Since the direction of the line as obtained from the map and this direction as laid off in the field are subject to considerable error, the first line run will probably pass to one side of the second controlling point. The distance by which this transit line misses its mark is noted, and the angular correction that must be applied to it is computed. The next line is run on the corrected direction and becomes the location line.

In running the preliminary line, stakes should be placed only at transit points. A straight line is the most difficult one to run, and particular care must be taken to avoid errors. Where changes of direction are necessary, the deflection angles should be carefully measured. It must be remembered that the preliminary is a reference traverse, and if the lines are not straight and the angles are not correct, the purpose for which it is run is defeated. The distances between hubs can be determined with sufficient accuracy by means of the stadia.

In country of the second class, preliminary lines are run over each of the routes chosen by the locating engineer on the reconnoissance survey. The data taken are such that the several routes can be compared with respect to distance, grades, and curvature. Usually lines run with the transit and stadia will give sufficient data as regards distance and curvature; and the elevations of enough commanding points can be taken with the stadia to indicate the grades that will be required by the several routes. From these

data, one of the routes will usually appear superior to the others, and what is generally known as a preliminary survey is then made over the selected route.

A preliminary survey, as generally understood, is a topographic survey of a narrow strip of country within which the road will pass. The purpose of this survey is to secure data from which a topographic map can be platted upon which the *paper location* is projected. The transit and stadia traverse already run over the chosen route is used as the base line from which the topography is taken. Although this line is usually measured with the tape, time used for this purpose is wasted, as it is impossible to plat the traverse distances to a consistent degree of accuracy. Spirit levels are then run over the line to determine the elevations of the transit points, and bench marks are established at half-mile intervals.

In country of the third class, several preliminary lines are required, each following one of the routes chosen by the locating engineer. The approximate grades of the lines between secondary controlling points are obtained from the data taken on the reconnoissance survey, and the preliminary line should be chosen so that the grade line thus determined will conform as nearly as possible to the surface of the ground. If this is done, topography will not have to be taken as far on each side of the line as would otherwise be necessary. In mountainous country, topography can be taken more accurately with the hand level than with the stadia, and line stakes are placed every 100 ft. and their elevations determined by spirit leveling, so that the hand level can be used. In other respects the methods of surveying in mountainous country are the same as those in country of the second class.

Railroad Location Survey

6. Location. The line is run very carefully on the location survey, stakes are driven every 100 ft., the plusses and angles of all property lines are taken, and curves are run in where indicated on the map. The distances from intersections with property lines to the nearest Government section monument, or other legal monument, are carefully chained, so that accurate descriptions of the right of way can be drawn up. All buildings near the line which will be damaged by it must be located and an estimate of the damage must be made. A level party follows the transit party, taking profile levels and establishing bench marks, unless these have been previously established. From these data, alinement maps, right-of-way maps, and

profiles are made; and after the grade line is established on the profile, construction can begin.

Railroad Construction Surveys

7. Reference Stakes. After the location has been made and accepted, and just before construction begins, the beginning and end of all curves and intermediate points on long tangents are "tied-in" by reference stakes so that after the construction work is completed these points can be relocated in their correct positions. There are several methods of referencing a point; the one shown in Fig. 1 is very satisfactory. A is the point to be referenced. First choose

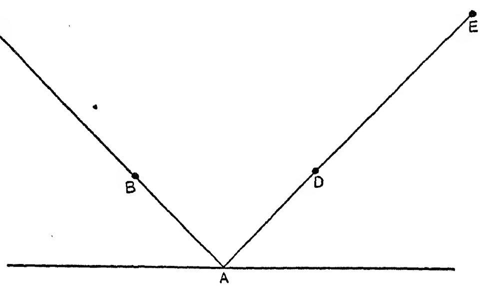


Fig. 1.

permanent points at C and E at least 300 ft. from A, making the angle CAE as nearly a right angle as possible. A distant windmill or house-chimney makes an ideal point. If these are not to be had, a nail driven into the trunk of a tree about 5 ft. from the ground is good. If hubs are used they should be made as permanent as possible, and located so that they will not be disturbed. After C and E have been determined, set a hub at point B on the line AC, and another one at D on the line AE. B and D should be placed far enough from the center line of the road so that they will not be disturbed during construction. Point A is relocated after construction by the intersection of the lines CB and ED, the transit being set up at B and D. Care and judgment should be used in locating reference points, as they *should be* used after the roadbed has settled for locating permanent monuments.

8. Slope Stakes. Slope stakes are then set at each station on each side of the center line at the points where the side slopes of the

cut or fill will intersect the ground surface. These stakes are for the guidance of the contractor, and have marked on them the vertical distance from the ground at the stake to the level of the roadbed. (For method of setting slope stakes, see section 239.)

9. Distribution Stakes. Distribution stakes are set to show the contractor the desired movement of the material from the cuts into the fills. The proper location of these stakes is explained in section 252.

10. Borrow-pit Stakes. These stakes are set to indicate to the contractor the limits within which he may borrow earth for making the fills.

11. Culverts and Trestles. The location of all culverts and trestles must also be staked out. If possible, these structures should be constructed in advance of the grading so that openings will not have to be left in the fill to be completed later.

12. Time. The time to set stakes is just before they are needed. If they are set too long a time ahead of construction, some of them will invariably be knocked out and will require resetting.

13. Monthly Estimate Surveys. During the last few days of every month, surveys are made to determine the amount of work which the contractor has completed during the month, so that he may be paid.

14. Finishing Stakes. From the slope stakes the contractor can construct the roadbed to within a few inches of the correct grade. At this stage of the work it is customary to give finishing stakes, which are stakes driven to grade at the edges of the roadbed at each station. From these stakes the contractor is able to finish the roadbed to the correct grade and width.

15. Center Stakes. After the earthwork is completed, the important points on the center line are relocated from the reference stakes; and center stakes, usually untacked, are driven from which the track is laid.

16. Grade Stakes. After the track is laid, grade stakes are driven at every station and at those points where the grade changes, with their tops to the grade of the final top or base of rail. Grade stakes are placed on the inside of curves, and the inner rail placed at grade. The outer rail is then superelevated the required amount above the inner rail.

17. Right-of-way Stakes. The right-of-way fences are usually built as soon as the materials for construction can be hauled over the line. For the guidance of the fence foreman, stakes are placed

on each right-of-way line (1) opposite the beginning and the end of all curves, (2) opposite each station on curves, (3) from 300 to 400 ft. apart on tangents, and (4) at all jogs in the right of way. These stakes should be long enough to be seen above weeds, wheat, oats, etc. Laths are excellent for this purpose.

18. Property Surveys. An exceedingly important part of the surveying operations is the determination of right-of-way and property lines and the legal description of the lands involved. This may be done as part of the preliminary and location surveys, but frequently special surveys are necessary.

Where the United States rectangular system of land surveys is employed, all lines should be tied to the Government monuments, and the descriptions should be made with respect to the legal subdivisions of land. Metes and bounds descriptions should be avoided whenever possible. Where the general land survey is by metes and bounds, this method must be used, and care should be taken that the surveys and descriptions are accurate.

19. Special Surveys. After the track is laid, special structures, such as station buildings, water tanks, cattle pens, etc., will require staking out. In addition, the parts of the right of way which are leased to coal, lumber, and grain companies must be staked.

Railroad Maintenance Surveys

Due to the fact that it takes two or three years for the roadbed to settle and for the track to become thoroughly embedded in the ballast, it will be necessary to reset center and grade stakes frequently during this period. All center stakes which are set after the track is laid should be tacked.

20. Monuments. After the track and roadbed have settled thoroughly, permanent monuments should be placed at the beginning and the end of all spirals and circular curves, between the branches of compound curves, and at intermediate points on long tangents. A section of steel rail about 3 or 4 ft. in length, embedded in the subgrade in the center of the track, with its top level with the ties and with a small drill hole to represent the point, makes an excellent monument. The monument should be free from contact with the ties and ballast to prevent its being moved by any shifting of the track.

21. Additional Tracks. Side tracks, business tracks, branch-line tracks, yard tracks of various kinds, cross-overs, etc., must be

staked out from time to time, as the need for such tracks arises. There is no end to surveys of this kind, and all the large railroads employ maintenance parties who do nothing else.

Organization of Parties

The field corps is usually divided into (1) a transit party, (2) a level party, (3) a topography party, and sometimes (4) a land-line party.

22. TRANSIT PARTY.—The members of the transit party and their duties are as follows:

The Locating Engineer is the chief of the entire surveying corps, and receives his instructions from and reports to the chief engineer of the railroad company. His duties are to direct all of the surveying operations from reconnoissance to location, to prepare the paper location, and to make all decisions relative to location, subject to review by the chief engineer. Frequently the chief engineer performs the duties of locating engineer.

The Chief of Party is next in rank to the locating engineer. His duties are to direct all the surveys, to provide accommodation for his party, to pay all general expenses, and in case a camp is necessary to purchase all supplies, and to manage the camp. On construction the chief of party usually becomes *Resident Engineer* and has charge of from 8 to 15 miles of construction.

The Transitman is next in rank to the chief of party, and in the latter's absence is in charge of the party. His duties are: to do the transit work, which consists of lining in the chainmen, measuring the angles between successive tangents, noting the bearings of the tangents, measuring the angles which the line makes with all railways, highways, streams, and property lines, and recording the plusses at which they cross the line; and to keep the notes of the transit party. The transitman frequently acts as chief of party.

The Head Chainman * ranks next to the transitman *in the transit party*, and is directly in charge of the rear chainman, stakeman, and axmen. His duties are: to see that the distances are chained correctly; to see that the stakes are driven on line, that they are driven straight, and that they are marked correctly; to direct the axmen where to cut in opening up the line; to set new transit points; and to direct the taking of plusses. The head chainman has a very impor-

* The term *tapeman* is frequently used since the steel tape has now replaced the old time chain.

tant position, as he regulates the speed of the entire party. In open country frequently the locating engineer takes this position. The head chainman carries the *zero end* of the tape.

The Rear Chainman's duties are: to hold his end of the tape on the last stake driven while the head chainman gets the distance; and to take and record all plusses which he turns over to the transitman at frequent intervals.

The Rear Flagman's duties are: to give the transitman a sight on the backsight station whenever he signals for it; and to carry excess baggage. The rear flagman *should be* a wide-awake man with good eyesight.

The Stakeman's duties are: to carry the stakes; to mark the station numbers on the stakes; and to drive the stakes as directed by the head chainman.

The Axmen do all the necessary clearing in order that the transit and level parties may have a clear path. They are sometimes required to make the stakes.

23. THE LEVEL PARTY.—The members of the level party and their duties are as follows:

The Levelman is chief of the level party and ranks next to the transitman in the surveying corps, when there is no regular topography party. His duties are: to run profile levels over the line and to establish bench marks; and to keep the level notes.

The Rodman's duties are: to hold the rod vertically upon the ground at each station, and at those intermediate points where the longitudinal slope of the ground changes; and to keep "peg notes" as a check on the levelman's computations.

24. THE TOPOGRAPHY PARTY.—The members of the topography party and their duties are as follows:

The Topographer usually holds equal or superior rank to the levelman in the surveying corps. This position is a very important one and should be filled by an experienced man. Frequently either the transitman or the levelman performs the duties of topographer. The duties of the topographer are: to take all necessary data for making an accurate contour map of a strip of country sufficiently wide to enable the engineer to make an intelligent projected location; and to record these data in such a way that they will be readily understood by the draftsman. The topographer is assisted in his work by one or more rodmen and tapemen.

25. THE LAND-LINE PARTY.—The duties of this party are: to measure the angles which the line makes with all railways, highways, streams, and property lines; to tie-in the line to the nearest Government monuments so that legal descriptions may be prepared of the required right of way; and to secure the names of the property owners. This work is highly important and should be done by an experienced man. Often, either the transit party or the topography party does the work of the land-line party.

26. Drafting. In addition to the above field parties there is the field draftsman, who does his work in camp. His duties are to plat the notes taken by all the parties the previous day, which necessitates the use of two sets of field note-books or else loose-leaf note-books. In some cases the draftsman, with the help of the locating engineer and the transitman, plats the notes each night; and the levelman plats the profile of the line over which he ran levels that day. Thus the map is kept up to date, and the locating engineer can project his location as the line advances.

Methods of Making Railroad Preliminary Surveys

The preliminary survey is the most expensive survey, and is of primary importance, since the location depends directly upon it. Hence the method of making it should be given considerable thought in order that it may be done with accuracy and economy. There are at present three general methods used: (1) the transit and tape method, (2) the transit and stadia method, and (3) the plane table method.

27. The Transit and Tape Method. This method is by far the most common. The transit party runs the line with transit and tape, the level party follows taking profile levels, and the topography party follows the level party; the land-line party may come in anywhere after the transit party. This kind of a survey will require from six to fifteen men, depending on whether the several parties have a separate personnel and on the number of axmen required to open the line.

28. The Transit and Stadia Method. This method consists in running the line with the transit and stadia. Stakes are placed only at transit stations, and the elevations of these points are determined by transit and trigonometric leveling. The location and elevations of important intermediate points along the line are determined in a similar manner. While the transit is at each station, the topography

around that point is taken with the stadia. Thus all the needed data are taken as the line advances. These data may be recorded in the note-book and worked up later by the party draftsman; but it is much better if the draftsman plats the notes as the transitman takes them and draws in the contours while the landscape is before him. The draftsman holds a very important position in this party and should be an expert in that line of work. A survey by this method is very accurate as regards the contours, which are the most important item on a preliminary map; and, if good men are employed, it is more efficient than the first method. This method requires a transitman, a draftsman, two rodmen, and as many axmen as the nature of the country may require.

29. The Plane Table Method. This method is very similar to the one just described and differs from it mainly in the use of instruments. The plane table takes the place of the transit, and the plane-tableman does the drafting. Owing to the difficulty in handling and setting up the plane table, it is doubtful whether this method is as efficient as the transit and stadia method; and although the services of the draftsman are dispensed with, yet the progress is possibly not so rapid.

30. Airplane Surveys. The development of the technique of making topographic maps from aerial photographs has made airplane surveys available for railroad reconnoissance and preliminary surveys. For the former, simple observation flights may suffice for selecting the feasible route or routes. For the latter, a regular series of photographs combined with some instrument work on the ground to establish points of control will often furnish sufficient data for the selection of the specific route, and even the preparation of the paper location.

The airplane survey is most valuable in rough, rugged country where the difficulty of making ground surveys is great. In such cases the airplane survey is vastly more rapid, and hence more economical.

31. Remarks. For long lines that justify the employment of a large number of men, the transit and tape method is probably the most efficient. In other cases, however, the transit and stadia method will prove the more economical. The latter method has not been used to any considerable extent, due to the fact that few engineers fully appreciate the advantages of the stadia method; but in the few cases in which it has been tried it has fully demonstrated its superiority, particularly for open country.

32. Bench Marks. When spirit levels are run over the line, bench marks should be established at half-mile intervals, approximately, and should be placed far enough from the center line so that they will not be disturbed during construction. After construction, permanent bench marks should be established on all permanent structures, such as concrete bridges, and at every station building along the line. It is extremely poor practice to use spikes driven into telegraph poles, mile posts, etc., as these are frequently moved and reset. If the road has few concrete or steel structures, then bench marks may be established on trestles. When these are renewed, the elevations of the several parts will rarely be changed more than an inch; whereas a bench mark on a telegraph pole may be changed several feet.

Note.—For further information on the subject of Railway Surveys and the Economics of Railway Location, see A. M. Wellington's "Economic Theory of the Location of Railways"; F. Lavis' "Railway Location Surveys and Estimates"; C. C. Williams' "Design of Railway Location," or W. L. Webb's "Economics of Railroad Construction."

HIGHWAY SURVEYS

Since a highway is a route of vehicular transportation very similar in operation to a railway, the general features of highway surveys are identical with those of railroad surveys. Such variations as do occur are due primarily to the difference in limitations placed on alinement and grades and to the fact that the majority of highway surveys are made for the purpose of improving existing roads.

In the days of horse-drawn traffic very sharp curves were permissible, but with the increased use of the motor car longer radii are demanded, and the practice in this respect is approaching that used on the railroads. Owing to the fact that each motor vehicle contains its own power plant, and also that the adhesion factor of the wheels to the road is much higher than for the locomotive, the permissible grades are much greater on the highways than on the railways. These factors give a somewhat greater flexibility in the location of the highway, especially in rough country.

One other factor must be kept in mind by the locating engineer of the highway, and that is the importance of appearance. On railroads only the enginemen see the various features as they approach them. Such of the passengers as see them get only a

glimpse from the observation platform as they recede. On highway, however, every traveler is looking forward, and every defect as well as every virtue is distinctly visible. Furthermore, the driver of a motor vehicle is dependent on the visibility of the roadbed and its structures for the safe handling of the vehicle, whereas the engine driver is relieved from the hazard of steering; hence in addition to good appearance, there must be safe sight distances.

Highway Reconnoissance Surveys

33. On New Location. Where a new road is to be laid out in new country, the reconnoissance survey is exactly like that for a railroad. The same methods are employed and the effect of the different classes of terrain is the same. Such roads may be entirely new routes or comparatively short stretches in existing highways where relocation is contemplated.

34. On Existing Roads. When the project is the improvement of an existing road, the object is to determine which of perhaps several possible routes is the most feasible. Each of the possible roads must be investigated. Such a reconnoissance is best made in an automobile equipped with an accurate odometer and a gradometer. One man drives the car and reads the instruments, while a second man makes sketches and keeps notes. A record is made of all such features as railroad crossings, stream crossings, steep grades, flood levels, entries into and routes through municipalities, availability of materials, etc., which may affect the cost of construction or the safe and convenient use of the highway after improvement. From these data the most satisfactory route can be determined.

Highway Preliminary Surveys

35. On *New Location.* When a highway is to be laid out on new ground, either entirely or as a relocation of part of an existing road, the preliminary survey is made in essentially the same way as the railroad preliminary. The same data are required and the same methods are employed. In only one respect is there any marked difference. The topography is usually more carefully taken, especially the cross-sections which are generally made with level, rod, and tape. This is because the information is employed not only for establishing the alinement but also for platting the cross-sections and for computing the amount of earthwork.

36. On Existing Roads. When an existing road is to be improved, a preliminary traverse is run to which all of the essential features of the existing road are tied. Care is taken to secure all information which may affect the nature or cost of the project and especially its relationship to existing improvements such as buildings, farm entrances, etc. Cross-sections are taken with the level and tape so that they can be platted and the earthwork determined.

Frequently old curves are to be abandoned and new ones of longer radii substituted. Each of these is essentially a short relocation and should be so treated. Especial attention should be given to the possibility of the elimination of jogs and slight deflections and other irregularities. The matter of appearance, as well as the serviceability of the improved road, should be kept constantly in mind.

Other Highway Surveys

37. Location and Construction Surveys. For highways these surveys are essentially the same as for railroads. Certain modifications may be required to suit the particular kind of road surfacing or the character of auxiliary structures employed.

38. Land-line Surveys. Since most of our highway rights of way consist merely of easements to the public for road purposes, the true title to the rights of way resting in the land owner, it is essential that the land lines be accurately determined, carefully described, and faithfully recorded as required by law. In so doing, and during construction, all Government monuments and other land-marks should be carefully referenced, reestablished, and preserved, and a complete record made of any changes made in the position or nature of the monuments.

39. Maintenance Surveys. As with the railroad, there is a perpetuity of miscellaneous surveys after the road is built for the purpose of maintenance, improvement, and additions. These are made at the time and in the manner suited to the particular needs.

Street Surveys

In a physical way, city streets bear much the same relation to the general highway system that yards and terminals do to a railroad system. From the administrative standpoint, however, the two are far different. On the railroad, both the main line and the yards

are under the same management, whereas the streets are under the jurisdiction of administrative bodies entirely separate and distinct from those controlling the highways, and hence the process of laying out and improving city streets is entirely different from the methods on rural roads.

40. Subdivision Surveys. Streets are laid out by the subdividers of the property. With the increasing adoption of city and regional plans and the extension of the powers of municipalities to control the subdivision of the land adjacent to them, it is highly important that subdivisions be properly made.

In flat country a simple boundary survey may be sufficient, including of course such things as streams and railroads which may affect the street plan or the use of the property. In more rolling territory a complete topographic survey should be made with care, since the streets must be adapted to the topography. The topographic map should be carefully preserved as it will prove of inestimable value in later improvements of the territory.

On the map of the territory, the paper layout or plat is made. This should be made after all features have been carefully studied. In some cases, especially on rough land, it is advisable to make the field layout before finally filing the plat. In any case the approval of the proper authorities must be obtained and the plat ultimately filed as required by law in the particular locality.

The ground layout should be carefully made. Permanent monuments should be placed at all principal corners and at such secondary corners as will make it possible to locate any lot or street line quickly and accurately. In general this means that all block corners should have permanent monuments, but it is not necessary to monument all lot corners, at least in the original layout. The descriptions of all monuments should be placed on the map and filed as part of the official plat.

The methods used on such surveys are transit and stadia or plane table and stadia for the topography, and transit and tape for the boundaries, with spirit-leveling where necessary.

41. Pavement Preliminary Surveys. No reconnoissance survey is made for street improvements. The street to be improved is chosen by the promoters, the city authorities, or by the property owners. Its position has already been established in the making of the subdivision. If the land is undeveloped, it may be necessary to retrace the street lines, but if the property is more or less built up the position of the street is usually clearly enough defined for

preliminary purposes without running a center line. This is especially true if sidewalks are in, as they are supposed to conform to the property lines.

If sidewalks exist, the measurements may be made on the walks, usually on one only. If sidewalks are not in, chaining pins or stakes may be used. If the streets are curved, some engineers prefer to run the center line, but property-line measurements are just as good, if correctly used. If the proposed pavement connects with an existing one, the measurement should begin at that end, otherwise there is no choice. Plusses to the nearest 0.1 ft. should be taken at all intersecting street and alley property lines, at all railroad and stream crossings, and at all other points where the natural topography or existing improvements may affect the design of the pavement. Cross-sections are then taken at intervals of 50 or 100 ft. as conditions suggest. In addition, the elevations of all features that may affect the grade of the pavement are determined. Bench marks should be established preferably in each block. The level circuit should be closed back over the bench marks to the starting point. The datum to be used is usually a city datum, and careful tie must be made to existing bench marks.

A convenient party for pavement preliminaries consists of four men, a chief of party, a levelman, and two rodmen. The chief of party and one rodman can chain the line, while the levelman and the other rodman pick up an existing bench mark and bring the levels to the street and establish the first bench mark on it. The chief of party should head the chaining, as his judgment should guide in tying-in the desired points. The party can then unite. The chief should keep notes, make sketches, and direct the work, while the levelman takes the cross-sections, using two rods, as this greatly expedites the work. Usually it is not necessary to use a tape on the cross-sections, because the distances are short and the rodmen can pace them. A reading is normally taken on the center line, at the gutter lines of the proposed pavement, and at each property line. This gives essentially five profiles of the street on the lines mentioned, and furnishes the data necessary to establish a grade line and to compute the amount of excavation.

Included in the preliminary should be a complete record of all drainage facilities, including manholes, inlets, tile lines, etc., with the depths to the flow line and the elevations of the tops, unless this information is available on other maps or plans. A record should also be taken of all other improvements, such as telephone and light

poles, sewer and other manholes, etc., which may affect the design or construction of the pavement, or which may require moving.

42. Pavement Location Surveys. From the data taken on the preliminary survey, a complete set of plans is made, including a well selected grade line. This grade line should be used in the field, unless some obvious change is required. It is poor practice to attempt to lay a grade line in the field.

The principal requirement for construction is a set of stakes, locating the curbs (or edges of pavement) and giving the grades. The best practice is to drive a set of "rough grading" stakes on each side of the pavement at intervals of about 50 ft. These stakes should be set back from the curb line from 5 to 8 ft. so as to be clear of grading operations. Level readings are taken on the tops of the stakes, and cuts to subgrade are computed and furnished the contractor. These stakes should be located with fair accuracy from the street lines.

After the rough grading is completed, exact line and grade stakes should be run for each edge of pavement or for each curb. If plain curb is used, these stakes may be about 2 ft. *inside* of either the back or the face of curb, as the contractor prefers, and driven to grade or some fixed distance above or below grade. If a combination curb and gutter is used, the stakes are best placed about 1 ft. *inside* the edge of gutter and to the elevation of edge of gutter. If no curb is used, the stakes should be about 1 to 2 ft. *outside* the edge of the slab and driven to grade.

Stakes should be placed at intervals of not more than 50 ft. on tangents and 25 ft. on horizontal and vertical curves. It is poor practice to set a few stakes and trust to the contractor to fill in.

43. Sidewalk Surveys. Frequently sidewalks are built before the pavements. In so doing it should be remembered that this fixes the pavement grade to a considerable extent, and consequently the sidewalk grades should receive careful attention. In flat country and on residence streets there is considerable flexibility between curb and sidewalk grades. On business streets, however, the sidewalks must conform both to the property and to the curb; consequently the entire design of both sidewalks and pavement should be worked out together, even if only one is to be built. In rolling country, where there may be considerable cross slope in the streets, the design of the grade lines for both walks and for the pavement should be made at the same time. The preliminary surveys should therefore be made accordingly.

44. Other Street Surveys. Lighting systems, storm sewers, sanitary sewers, water and gas pipes, etc., are frequently placed in the streets. These should receive consideration both in making the subdivisions and in fixing the street widths so that there may be ample room. The individual survey for each of these improvements follows the ordinary methods for such work.

CANAL AND DRAINAGE SURVEYS

45. Canals. A canal is an artificial waterway for the purpose of providing a means of transportation, or for the purpose of conducting the water itself to desired points. A drainage ditch is a canal for conducting away excess water.

A drainage ditch or canal must be kept within such grade limits that adequate flow is maintained without developing excess velocities which may cause erosion. The curves also must be sufficiently flat so that the banks will not be seriously eroded. A canal for navigation must have an alinement such that the type of vessel using it can be easily and safely navigated. Usually the flow of water is small, and frequently intermittent, due to the operation of the locks necessary to overcome differences of elevation.

It is thus seen that any canal is quite restricted as to grade, and this has a serious effect on the location, although there may be wide flexibility in alinement. The surveys for canals must therefore take cognizance of these limitations, and this may modify to some extent the surveys as outlined for railroads.

The reconnoissance may be exactly the same as for a railroad. A drainage canal may follow a stream, and a study must be made of the stream as to possible relocations and cut-offs. A transportation canal must be supplied with water for operation, and the determination of possible sources is part of the reconnoissance.

Preliminary surveys are very similar to those for railroads. A line is run in the same way, and topography is taken in the same way, together with such other data as may affect the location or design of the canal.

Location surveys are essentially the same, except that offset lines must normally be used, because, unlike on the railroad, the center line can not be used for the final work, since the canal will be filled with water. The same is true of construction surveys and maintenance surveys. In none of these is there anything of unusual note, if due consideration is given the characteristics and purposes of the particular project in hand.

PIPE-LINE SURVEYS

46. Character and Purpose. Pipe lines as here considered are pipe, extending through open country, often for long distances, for the purpose of transporting liquids, usually petroleum or water.

Oil pipe lines are usually of comparatively small diameter, from 8 to 12 in., and extend long distances. They depend on pumps for forcing the oil through the pipe, and consequently they are very flexible as to alinement and grade. Water pipes on the other hand may reach several feet in diameter and often depend on gravity for flow. With large sizes, the curves must be comparatively easy, and for gravity flow the grades must be suitable, usually all in one direction, but not necessarily so, because an effective hydraulic grade can be maintained even if the actual grade dips below it and rises to it again.

Pipe lines are usually placed under-ground. This often permits the continued use of the ground above for ordinary purposes, especially for agriculture. The right of way, therefore, consists merely of an easement for the construction, maintenance, and repair of the line. Frequently the easement is leased on an annual rental basis.

These factors are of importance as they have an effect on the character and extent of the surveys.

47. Surveys Required. A reconnoissance is necessary to choose the practicable routes. This is similar to the railroad reconnoissance. A preliminary survey then follows, which may be very similar to the railroad survey in case of a large water line on gravity flow, but may be a simple line for a small size oil line. Sometimes the preliminary and the location survey for an oil line are combined. In both cases great care must be taken with the land-line surveys, so as to make it possible to prepare adequate descriptions and contracts for the right-of-way easements, and to locate the line accurately so as to remain on these easements. Line and grade stakes are required for construction, and occasionally where deep cuts are made, slope stakes may be needed.

Pipe lines may require special surveys, where they cross large streams or where tunnels may be employed. These are, however, similar to railroad surveys for the same purpose, keeping in mind the essential characteristics of the project.

TRANSMISSION-LINE SURVEYS

48. Character and Purpose. Transmission lines are built for the purpose of transmitting electric power or for conducting communication by telephone or telegraph. Power lines may differ considerably from telegraph and telephone lines in their details of construction, but all are essentially the same in the surveys required.

In most cases transmission lines are carried overhead on poles or towers. This permits the use of the ground for other purposes, except for the area immediately around the towers or poles. Here again rights of way are easements similar to those for pipe lines. The poles and towers, however, permanently occupy the land, whereas the space between is needed only during construction and when repairs are made.

Transmission lines are very flexible in line and grade. Right-angle turns may be made at a single pole or tower, or curves may be used if necessary. In general, changes of direction are desirable at a minimum number of poles or towers on account of lateral bracing. A single tower at an angle would, therefore, be preferable to several towers, making a curve. The grades can have nearly any value, depending on the clearance between the wires and other objects in the spans.

Sometimes transmission lines are placed under-ground. In these cases, they can be considered as pipe lines so far as the surveys are concerned.

Accessibility is an important factor in the location of transmission lines. For this reason telephone and telegraph lines are frequently located along highways, and the poles are placed on the highway right of way. Power lines may parallel highways, but they should be far enough from them so as not to induce electric currents in existing telephone and telegraph lines.

As far as possible, woods and forest should be avoided, because of the expense of clearing and of maintaining. Also, power lines should be a safe distance from all houses.

49. Surveys Required. The surveys for a transmission line are almost identical with those for pipe lines. A reconnoissance survey is made to choose the practicable routes. More detailed surveys are made to establish tower locations, take care of stream crossings or steep inclines, and provide for other special features. Suitable sites for substations or repeater stations must be considered.

Location surveys and land-line surveys are in general the same as for pipe lines.

CHAPTER 2

MAPS, PLANS, AND PROFILES

THE location, construction, and maintenance of any route of transportation require the preparation of a large number of drawings which may be broadly classified as follows:

1. *Maps*, which are horizontal projections of a considerable portion of the route, showing the essential features.
2. *Plans*, or comparatively large scale drawings of special features of the route, or more often of specific structures such as bridges and buildings. The term *plans*, however, is frequently applied to the detailed horizontal projection of a highway or street (the paper location) and the term *details* used for the drawings of specific structures.
3. *Profiles*, which are essentially vertical projections corresponding to the maps. *Cross-sections* are short profiles transverse to the main direction of the route.

Railroad maps, plans, and profiles are typical of all route drawings and therefore will be considered first.

Railroad Reconnoissance Maps

50. Reconnoissance Map. This map consists of a general sketch of the country which the locating engineer has investigated, and shows the several routes that are possible. Only controlling points, such as towns to be passed through, available stream crossings, saddles in the ridges, etc., are shown. If an existing map of the country is available, it is best to draw in the routes directly upon it. The notes taken on the reconnoissance survey regarding the geological formations, the cultural features of the country, the width, depth, and current of streams, etc., are considered part of the reconnoissance map, and frequently have great weight in the choice of route.

51. Reconnoissance Profile. This profile is made from the elevations of the controlling points and the distances between them

Profiles are made for each of the several routes and are frequently placed on the same sheet, so that a more intimate comparison can be made. The purpose of the reconnoissance maps is to eliminate the impracticable routes and to determine which ones will bear a more detailed investigation.

Railroad Preliminary Maps

52. First Preliminary Maps. These maps show the transit and stadia lines run, and all railroads, highways, and streams that cross the lines. Profiles corresponding to these maps are made from the elevations of the traverse stations. From a study of these maps and profiles, one route—occasionally two—will appear superior to the others, and the preliminary survey proper is then made over this route.

53. Preliminary Map. The preliminary map proper is generally made to a scale of from 200 to 400 ft. to the inch, and is a complete topographic map of a strip of country from 100 to 1000 ft. in width. All highways, railroads, streams, buildings, and other features that may affect the location and construction of the line are shown. Frequently it is desirable to show the property lines, the names of land owners, and Government or other land monuments. Contours are an essential part of this map. The usual contour interval is 5 ft., but in very rough country 10 ft., and in very flat country 2 ft. may be used. All geological formations and other similar features that might affect the location of the line are also indicated. Occasionally, special conditions may require an auxiliary map to a scale of 50 or 100 ft. to the inch over short stretches of the line.

54. Projected Location. The projected location, often termed the *paper location*, is made on the preliminary map. The line which will give the best alinement and grades from the standpoint of operation is first projected, and this is the one to be used, unless the cost of construction is prohibitive. In case the cost is excessive, a line whose construction will come within the allowable cost, and at the same time keep within the allowable limits of curvature and grade, is laid out. This can be done only by trial and requires a large amount of skill and judgment. After the tangents have been projected, they are connected by curves which most nearly conform to the contour of the ground, and at the same time keep within the maximum curvature. The entire alinement is then reviewed, and

changes made in tangents or curves as may appear desirable, until a satisfactory alinement is obtained. If the curves are to be spiraled, allowance must be made at this time.

In order to determine the grades and the amount of earthwork for the several projected lines, it will be necessary to construct a profile for each of them from the contour map. In locating grade lines on these profiles, it must be remembered (1) that intersecting railroads must be crossed either at grade or at a clearance distance above or below grade, (2) that highways can be raised or lowered within certain limits, and (3) that streams must be crossed a safe distance above high-water marks.

The projected location is platted in detail on the preliminary map and a profile of the projected line is constructed from the contours, and a tentative grade line is established on the profile.

55. Location Notes. Notes are then made from the accepted projected location. The bearings of tangents, the plusses of the beginnings and the ends of all curves, the central angles of all curves, and the degrees of curve are scaled off the map and recorded. These condensed notes are used in making the final location.

Railroad Location Map

56. Location Map. The location map is usually made to the same scale as the preliminary map, and shows all railroads, highways, streams, and property lines that cross the line, together with their plusses; the names of the property owners and the amount of right of way required from each; the Government subdivision lines and the numbers of sections, etc.; the distances from the line to Government monuments; the boundaries of each field through which the line passes; the location and size of all stream openings; and any other data that will be of use to the construction engineer or to the right-of-way agent. If the line is located in territory not under the United States rectangular land-survey system, the line must be tied to property lines and to such land monuments as exist.

The location map can often be made by adding the necessary data to the preliminary map, and then making a new tracing, omitting the contours and such other features as are not desired on the final map.

57. Location Profile. A profile is made from the levels which are run over the located line, and shows: the ground line; the grade line,

including the per cent of grade of the different portions and the elevations of all points where the grade changes and of all stations on vertical curves; the location and dimensions of all openings in the embankment; the plusses of railroads, highways, streams, and openings in the embankment; the elevations of the top of rail of all intersecting railroads; the crown of roadway of highways or streets that pass overhead or underneath, and the high-water marks of streams; the descriptions and elevations of all bench marks; a rectified alinement map at the bottom of the profile sheet, containing practically all the data that are shown on the location map; the earthwork distribution diagram, which is drawn between the profile and the alinement; on the profile, the economical movement of the earth from the cuts into the fills; and the amount of excavation, overhaul, and borrow at all points along the line. In fact, the location profile contains practically all the information needed by the resident and construction engineers. It is used in staking out all the construction work, with the exception of buildings, bridges, trestles, and other structures for which separate plans are made.

Additional profiles are made for any change in the elevations of highways and railroads crossed by the line. These profiles are short and are sometimes placed on the location profile opposite the points where the changes are made. Cross-sections for embankments, cuts, borrow pits, or spoil banks are made as the need arises.

Railroad Right-of-Way Maps

58. Right-of-Way Maps. These maps are of two kinds, legal maps and maintenance-of-way maps.

Legal right-of-way maps are made on sheets which are the same size as the sheets on which the conveyance deeds are drawn up, usually $8\frac{1}{2}$ by 13 in. A separate map is made for each description, and a blueprint is attached to the deed and becomes a part of it and is recorded along with the deed. The railroad company usually preserves its copies of the deeds along with the right-of-way maps in the form of a book, which is known as the right-of-way book.

Maintenance-of-way right-of-way maps are made from the location map; and a separate sheet, about $8\frac{1}{2}$ by 10 in., is used for each section (that is, Government section) or for each mile of track. These sheets are bound into a book and are used by the maintenance-of-way department.

Railroad Construction Maps

59. Station Maps. These maps are usually made to a scale of 50 to 100 ft. to the inch and show the proposed buildings and tracks at each station. Any changes made in the plans during construction are recorded on the map. Some railroads have standard plans for station layouts, and these are followed as closely as conditions will permit. The station maps are accompanied by the plans and details of the buildings and other structures required in the station layout.

60. Progress Profiles. Each month, after the monthly estimate surveys have been made, the amount of grading done during the month is shown graphically on the location profile in colored pencil. Each month a different color is used, so that the progress of the construction work can be seen at a glance, and the amount done each month compared with that of preceding months.

61. Progress Photographs. For the same reasons, photographs should be taken each month, or oftener, of the various structures under construction, such as arches, bridges, trestles, station buildings, etc. Photographs are excellent auxiliaries to a written report.

Highway Reconnoissance Maps

62. Reconnoissance Maps. When the highway is to be located through new territory, the reconnoissance map is practically identical with the railroad reconnoissance map.

When the highway consists principally of the improvement of an existing road, the reconnoissance map frequently takes the form of the *strip map.* This is a map on a scale of about one inch to the mile, and shows a strip of land along the proposed route from 4 to 6 miles in width, or enough to show the several alternate routes under consideration. On this map are indicated the various routes, the one which seems most desirable being most prominently shown. Notations of the items recorded on the reconnoissance are made on the map. Profiles may or may not be platted from the reconnoissance notes as the needs appear to indicate.

Highway Preliminary Maps

63. Preliminary Maps. When the proposed highway is through new territory, the preliminary map becomes the exact counterpart of the railroad preliminary map. The same data are platted, and

the line is projected in the same manner, taking into consideration the different limits placed on grades and curvature. One difference, however, is that the cross-sections are platted to scale and are used in conjunction with the profile in establishing the grade line.

When the road is essentially the improvement of an existing highway, the preliminary map becomes a study sheet for the working out of the construction plans. The data taken on the preliminary survey are platted to the scale of the finished plans, which is usually 100 ft. to the inch. A profile with the same horizontal scale and a suitable vertical scale for the given territory is also drawn. The cross-sections are platted on separate sheets to suitable scales, usually 10 ft. to the inch both ways.

The horizontal alinement and the grade line are then worked out as before, and the necessary changes in right of way, etc., are determined and shown on the map.

Highway Construction Plans

64. Construction Plans. A location map is rarely made for a highway, unless the location is for a considerable distance through new territory. In general, the location and construction drawings are combined on a series of plan-profile sheets more nearly corresponding to the location profile of the railroad.

To facilitate the administration of the Federal Aid Law, the Bureau of Public Roads devised the so-called Federal Aid Sheets and required their use on all federal-aid projects. These sheets have been almost universally adopted for highway construction drawings. The sheets are about 24 by 36 in. in outside dimensions and are ruled complete on each sheet. Spaces are provided for identifications, while the main ruling is made in three styles as follows:

1. Half-plan-half-profile sheets, on which the upper half is blank to receive the plan and the lower half is ruled for profiles. The horizontal divisions are one-half inch and the vertical divisions are one-tenth inch.

2. Quarter-plan-quarter-profile sheets, on which the upper quarter is plain, the next quarter below is profile-ruled, the next quarter is plain, and the lowest quarter is profile-ruled. This sheet is used for straight alinement where grades are light, as it permits twice as much line to be shown on a single sheet.

3. Cross-section sheets, which are ruled to one-tenth inch in both directions.

The detailed horizontal alinement, together with all of the features needed on a general construction plan, are shown in the plan areas. The profile shows the grade line, earthwork distribution and balance points, and the other features as described for the railroad location profile, except that the distribution diagram is not shown. If this is used, it is made on separate sheets.

In addition to the plan-profile sheets, cross-section sheets, details of structures, etc., must be made. A complete set of road plans will thus consist essentially of the following:

1. A cover sheet showing the general location and title of the project and certain general data.
2. A series of plan-profile sheets as described.
3. Sheets showing the standard cross-sections of the completed roadway.
4. Detailed drawings of bridges, culverts, drainage appurtenances, and other structures.
5. The cross-section sheets used in determining the earthwork.

During construction, progress sheets showing the work completed each day, week, or month are made. These are similar to the railroad progress profiles, but may also include a progress plan. Progress photographs are as valuable on highway work as on railroad work.

Street Plans and Profiles

65. Plans and Profiles. Since no reconnoissance is made, there is no reconnoissance map on street work.

The preliminary map is merely a study sheet for the establishment of the horizontal plan and the grade line, and is rarely preserved. Since most pavement work is paid for by special assessment, it is usually necessary to file the plans for record along with the ordinance providing for the work. Only the finished drawings are filed.

The completed set of street pavement plans is therefore very similar to the highway plans, and consists of the plan-profile sheets, the details of drainage appurtenances, curbs, etc. Cross-section sheets are usually not included, and the cover sheet is often omitted, except on very large projects.

Where sewers and other underground work, sidewalks, etc., are included in the project, their positions are shown on the plan, and

their grades are shown on the . . sidewalks, etc., are similar to the foregoing.

Highway and Street Right-of-Way Maps

66. Right-of-Way Maps. The title to street and highway right of way is vested in the public. The descriptions are therefore required to be filed for public record, as provided by law in each locality. Sometimes the records are mere descriptions, but more frequently plats of greater or less completeness are included. Since a public record is required, dependence is made upon it, and local road and street authorities are often very lax in keeping records of their own. Moreover, many of the records filed are very poor, consequently highway and street rights of way are frequently in doubt.

Existing rights of way have been provided by dedication, purchase, or condemnation. New roads and streets are opened, or additional right of way is provided in the same way. The records should be full and complete, and each local authority having jurisdiction would do well to make records similar to the right-of-way maps of the railroads. These should be in duplicate, one set filed as required by law and the other set kept for local record.

Subdivision Plats

67. Subdivision Plats. When a new subdivision or addition to a city is laid out, a map of the tract of land showing the lots, blocks, street, alleys, etc., must be prepared and filed with the proper authorities. It then becomes the official record and the basis of the legal descriptions of the various subdivisions of land contained in it.

Extreme care must therefore be taken to make the plat complete and accurate. All dimensions should be carefully checked to see that no errors creep in and that there are no ambiguities in the layout or in the dimensions. It is a rule of land surveying that distances take precedence over angles. Distances should, therefore, be provided wherever possible, but, in case angles are desired to govern, distances which might supplant them should be omitted.

All Government or other existing land lines should be shown, and any monuments on them which control the subdivision should be carefully noted and, if possible, referenced so as to be relocated

easily, if they should happen to be disturbed. New monuments, established as part of the subdivision, should be accurately located, described, and referenced.

In general, contours are not placed on the record plat. A separate map, made on the legal plat as a base, should be used for showing the topography. A copy, however, should be filed with the city engineer, as it will prove of great service in planning pavements, sewers, etc.

Canal, Pipe-line, and Transmission-line Maps

68. Miscellaneous Maps. Since the general scheme of the location, construction, and maintenance of canals, pipe lines, transmission lines, etc., is very similar to that of a railroad, the maps and other drawings required are the counterpart of those required by the railroad, keeping in mind the peculiarities of the particular type of route under consideration.

Reconnoissance, preliminary, location, construction, progress, and maintenance maps and profiles will usually be required as well as plans and details of special structures. These drawings should be clear and well executed and made to scales suitable for the particular work.

CHAPTER 3

DISTANCE, CURVATURE, AND GRADES

There are three principal factors which the engineer must consider in the location of any route of transportation, if the project is to be an economic success. These factors are *distance*, *curvature*, and *grades*, and their importance is due to their influence on (1) the cost of construction, (2) the cost of maintenance, and (3) the cost of operation. It must be remembered that the first cost is a fixed amount, whereas the costs of maintenance and operation continue as long as the route exists, and consequently the additional cost of operating and maintaining a poorly located line will in time offset any saving that may have been effected in the cost of construction.

These factors have certain normal characteristics which are briefly discussed in the following sections. It must be remembered, however, that special conditions may modify or displace the basic characteristics for the time being. Such special conditions must receive careful consideration, but the fundamental ideas must not be overlooked.

Distance

Other things being equal, the ideal alinement would be straight between controlling points. Such an alinement rarely exists, because in some cases it may be actually impossible to build a straight line, and in most cases the first cost will probably be prohibitive, due mainly to excessive earthwork, unfavorable bridge sites, expensive right of way, etc. It is normally necessary, therefore, to deflect from the direct line and by so doing introduce additional distance. Again, the grade of a direct line may be too steep for economical operation, and this can be corrected only by the process of *development*, or the introduction of extra distance in order to reduce the grade. Furthermore, under certain conditions, the amount of grading may be so reduced by increasing the distance that the longer line will actually be cheaper to build than the shorter line. The problem, therefore, is to find a line on which the saving in the cost of construction, due to less grading or more favorable special

features or the saving in operating cost on a flatter grade, more than compensates for the increased cost of construction, maintenance, and operation due to the greater length of route.

69. Construction. Exclusive of special features, such as large bridges, tunnels, etc., the cost of construction of any route is practically proportional to its length. Normally, therefore, any additional distance adds proportionally to the cost of construction. Of course any unusual item introduced by the additional distance must receive due consideration.

70. Maintenance. In general, the maintenance costs of a route are also directly proportional to the length, and this view is sufficiently accurate for the locating engineer when considering the advisability of introducing additional distance.

71. Operation. Operating costs also increase with the length, but the increase may not be directly proportional to the increased distance. This is especially true on railroads where the cost of operating small additional distances is only about one-third as much per train-mile as the average train-mile cost over an entire engine division. The cost of the additional distance then gradually increases until it reaches the average cost, which occurs when the additional distance is still well under the length of an engine division. The present tendency to increase the length of engine divisions also tends to decrease the average train-mile costs and at the same time tends to reduce the distance in which the cost of additional distance equals the average cost. Commercial canals show similar characteristics in the relation between operating costs and distance.

It has been found that the average cost of operating individual motor vehicles on highways is practically proportional to the distance run. Although this fact may not be exactly true for traffic taken collectively, it is sufficiently accurate for ordinary use and is generally so taken. The same fact is essentially true for pipe and transmission lines.

The increased cost of operation due to added distance should not be confused with, or neglected on account of, a very real saving in operating costs which may result from decreased grades obtained by development. In fact the economic limit of development is reached when the added costs due to increased distance are just equal to the decreased costs due to flatter grades.

72. Revenue. Distance is sometimes closely related to the revenue to be obtained from commercial projects such as railroads, or to the service to be rendered by public projects such as highways. For

example, it may be found that, by deflecting from the direct line, additional traffic and hence additional revenue can be obtained by the railroad, or that additional population can be served by the highway. The locating engineer must then balance the increased cost of construction, plus the perpetual costs of maintaining and operating the additional distance, against the probable increase in revenue or the improved public service. This often involves the question as to whether the main line should be deflected through the new point, or whether the main line should be made direct and the desired point reached by a branch. This must be studied as a local problem. In general, trunk highways are finding it more economical to keep the main line direct and to use a branch, even for short distances, rather than to deflect the main route. This may be somewhat less true of railroads.

Curvature

Aside from the fact that the presence of curvature indicates deviation from a direct line, with the introduction of additional distance, curvature itself affects the three major items of cost.

73. Construction. Curvature adds to the cost of construction on practically all routes. Due to superelevation and to drainage requirements, both railroads and highways are more difficult to construct on curves than on tangents. On railroads, too, it may be necessary to compensate grades for curvature as explained in section 77, and this may add indirectly to the cost of construction by adding to the earthwork. Transmission lines must make provision to balance side pull on curves and this adds to the cost. Canals and pipe lines may, however, show little or no increased cost of construction on curves.

74. Maintenance. Railroad track is more difficult to maintain to line and surface on curves than on tangents, and there is also increased wear on rails, ties, and ballast, all adding to the costs of maintenance. Records seem to show that highway curves are only slightly more expensive to maintain than tangents when the surface is paved, but when the surface is of gravel or stone the maintenance cost on curves is high. Erosion of the banks on curves adds to the maintenance costs of drainage canals. Barge canals may show no increase, because of the low velocity of flow. Pipe lines also show little increase in maintenance due to curvature, whereas transmission lines may have considerable increase, because of the special provisions required by side pull. This may be reduced

in transmission lines by making abrupt angles in the alinement instead of regular curves.

75. Operation. Curvature appears to have little or no effect on the cost of operating canals and transmission lines, nor on pipe lines when the changes of direction are made by gradual curves and not by abrupt bends.

So far it has not been possible to determine whether curves add directly to the cost of operating highways or not, and therefore it is usual to assume that they do not. Curvature, especially if sharp, may so reduce speed as to limit the capacity of the road, cause traffic congestion, or add to the hazards of traveling. Although these conditions can not be evaluated in money they are important items and must receive due consideration by the locating engineer.

On railroads, however, it has been demonstrated that curvature adds materially to the cost of operation, due especially to increased resistance to train movements. The effect of this will be briefly discussed in the following sections.

76. Railroad Curve Resistance. The wheels on engines and cars are rigidly attached to the axles. Consequently, in rounding curves both wheels must make the same number of revolutions, although they travel different distances, because of the difference in length of the inner and the outer rails. This results in a certain amount of slippage of the wheels, which adds to the tractive resistance. Apparently this resistance does not increase directly with the degree of curve, but the rate of increase becomes less as the curves become sharper. In addition to the foregoing, if the train does not run at the speed for which the curve is superelevated, there is an additional pressure of the flanges on the rails, and this adds to the tractive resistance. The variation in resistance with speed is not large and is generally neglected.

Values of the curve resistance have been determined experimentally, from which the following practices have been adopted:

Curves 0°–8°:	0.8 lb. per ton or 0.04% grade, per degree.
Curves 8°–15°:	0.6 lb. per ton or 0.03% grade, per degree.
Curves 15° up:	0.4 lb. per ton or 0.02% grade, per degree.

The resistance at starting is about twice these values and hence should be considered if stops are required on curves. For short curves the resistance should be taken only for the proportional length of the trains.

77. Compensated Grades. From the standpoint of operation, the effect of a curve is to increase an ascending grade on which it occurs

by the amount of the curvature resistance. Thus a 6° curve on a level grade would be equivalent to a grade of $6 \times 0.04 = + 0.24\%$ on straight track; and on a $+1.00\%$ grade it would be equivalent to a $+1.24\%$ grade on straight track.

It is therefore evident that if a maximum operating grade has been chosen, the actual grade must be reduced by the amount of the curve resistance. This is termed *compensating for curvature*, and the grade is spoken of as a *compensated grade.*

For example, if a maximum operating grade of 2.00% is chosen and a 10° curve occurs on it, the actual grade must be $2.00 - (10 \times 0.03) = + 1.70\%$, and this is the grade to be established on the profile over the length of the curve. This grade would be termed a "2% compensated grade."

On descents, the effect of curvature is to reduce the slope numerically. Thus for descents, the operating grade in the above example would be $-1.70 + 0.30 = - 1.40\%$. This condition, however, is normally favorable to traffic and is usually not considered.

78. Choice of Curvature. Curves may limit the speed of traffic on railroads and highways. Formerly only passenger trains were considered as being affected by this limitation of speed. At present, however, freight trains are being operated at express speeds and freight service is therefore demanding as easy curves as passenger service. On highways the increasing speed of motor vehicles, both passenger and freight, with the resulting necessity of greater sight distances and easier control of the vehicle, is also demanding flat curves.

The total resistance to train movement is directly proportional to the total change of direction and varies only slightly, as previously shown, with the degree of curve. Hence the important thing on railroad location is to keep the total curvature (sum of the central angles, I) as low as possible. In the large amount of railroad realinement work which has been done, decreased total curvature has always been a primary object, with increased radius a secondary consideration.

The total central angle is of little importance on highways as far as cost of operation is concerned. But, because of the need for ample sight distances and ease and safety in driving, a minimum of curvature, combined with flat curves, is highly desirable. Consequently, on both new highways and the relocation of old ones, increasing attention is being given to reducing the total curvature to a minimum.

Total central angle appears to be of little moment on canals, pipe lines, and transmission lines. Flat curves are desirable on all of

these, except transmission lines, where often the entire change of direction can be made more economically at one or two special towers or poles, which are designed to take the side pull resulting from the tension in the wires.

Railroad Grades

In flat, prairie country grades are so small that they play little or no part in the choice of location, but in rough country grade is the most important of the three factors which affect the choice of route for a proposed railroad, and is the one which requires the most study by the locating engineer. The effect of grade upon the costs of construction, maintenance, and operation will be discussed separately, as in the case of the preceding factors.

79. Construction. The cost of construction is directly affected by the grades adopted. If steep grades are used, the cost of construction may be no more than would be the case in more level country with flatter grades. On the other hand, flat grades in rough country mean high fills and deep cuts, which increase the cost of construction tremendously. The question before the locating engineer in comparing two routes, or in comparing two sets of grades on the same alinement, is whether or not the greater cost of grading on the line with the flatter grades is justified by the saving in maintenance and operation expense. This is purely a question of economics. The difference in the cost of grading is easily computed, and the saving in maintenance and operating costs can be estimated with a reasonable degree of accuracy, as explained in subsequent sections.

80. Maintenance. The maintenance items affected by grade are renewal of ties and rails and increased depreciation of locomotives and cars. It is found that the life of ties is somewhat less on steep grades than on level grades; also, that the wear on rails is greater, due to the slipping of wheels, to the use of sand to obtain traction, and to the greater number of engine-miles required because of the larger number of trains necessary to haul the same total tonnage. The wear on the drivers, car wheels, brake shoes, and drawbars is also greater on grades than on level track. The increased cost of maintenance, however, is not large. Unless the grades are steep enough to require the use of brakes in descending, the additional cost is practically negligible. Where brakes are used, Wellington estimated the additional cost on the items mentioned to be about 5%.

81. Operation. The cost of operating a railroad with steep grades is very great. The subject of locomotive performance is so

closely related to the cost of operating grades that it must be considered before the cost of operation can be discussed further. The resistances to be overcome by the power of the locomotive are tractive resistance, grade resistance, and curvature resistance.

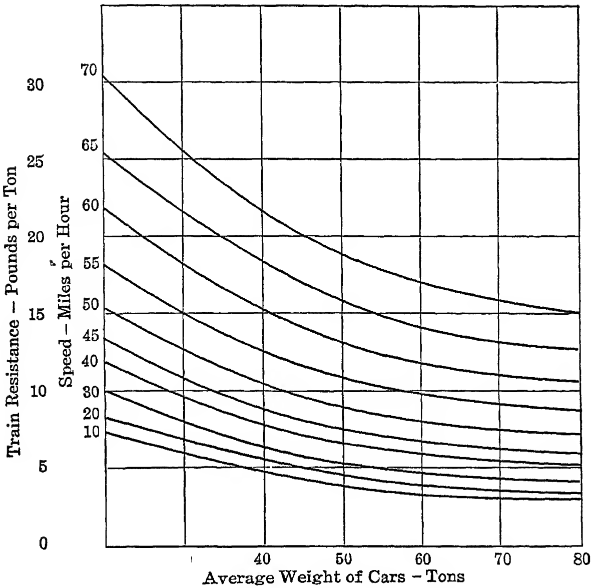


DIAGRAM A.—Freight Train Resistance. Data from *Bull.* 376, University of Illinois Engineering Experiment Station, 1948.

82. Tractive Resistance. The resistances to be overcome on straight level track are (1) inertia resistance, or the resistance to be overcome in starting or in accelerating a train; (2) rolling resistance, caused by the friction of the wheels against the rails; (3) journal friction, caused by the friction of the axles in the journals; (4) atmospheric resistance, caused by movement through a still atmosphere; and (5) oscillation resistance, due to uneven track. It is not considered necessary to discuss each of these forms of resistance separately, but all of them can be combined into a single item which may be termed *tractive resistance.* Diagram A gives the total tractive resistances on level track for speeds from 10 to 70 miles an

hour and for trains of loaded cars weighing from 20 to 80 tons each. The resistance caused by head winds and wet, slippery rails is not included in the values given. Diagram B gives the tractive resistance for steam and Diesel locomotives at various speeds.

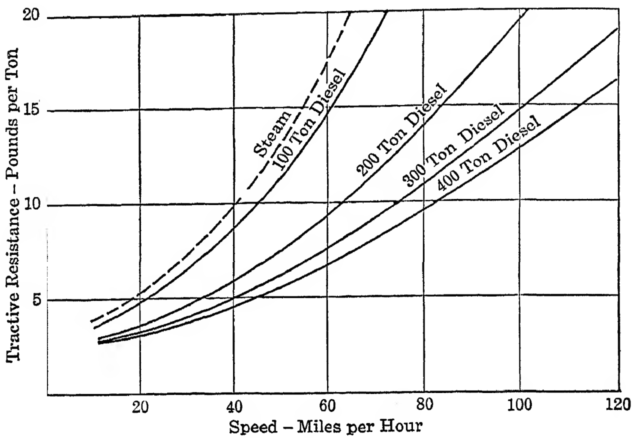


DIAGRAM B.—Locomotive Resistance. Data from Electromotive Division, General Motors Corporation.

83. Grade Resistance. The grade resistance can be determined with mathematical exactness and is 20 lb. per ton for each 1% of grade. Thus it is seen that the resistance offered by a 1% grade is about five times the resistance on a level track, and the importance of grade resistance is obvious.

84. Curve Resistance. If a curve comes on a grade, there is an additional resistance to overcome, unless the grade is compensated for curvature. The subject of compensation for curvature and curve resistance has already been considered in sections 76 and 77, and therefore a discussion here is unnecessary, further than to say that, if the grade is not compensated, the curvature resistance must be added to the tractive resistance and the grade resistance to obtain the total resistance to be overcome by the locomotive in hauling a train up the grade.

85. Tractive Effort. The tractive effort of a locomotive is the total driving force delivered at the rims of the drivers. Its max-

imum value depends on the weight on the drivers and the coefficient of friction between the wheels and the rails.

On clean, dry rails the coefficient of friction is 0.33 to 0.35, but this can be increased to about 0.40 by spraying fine sand between the wheels and the rails. On wet or frosty rails the coefficient may drop to 0.20 or less, but this "slipperiness" may be largely overcome by the use of sand.

Since the driving torque of a steam locomotive is pulsating, such engines are designed for a coefficient of friction of 0.25 to 0.28 so that the drivers will not slip at low speeds at the peaks in the torque. With electric traction the torque is smooth; hence such engines may be designed for coefficients of friction of 0.30 to 0.34.

The *drawbar pull* is the *net* tractive force delivered at the drawbar and is the force available for moving the train. It is equal to the tractive effort minus the total force required to move the locomotive itself.

86. Steam Locomotives. Basically, a steam locomotive consists of a boiler supplying steam to a duplex engine with double-acting cylinders which turn the drivers. The crank-pins on opposite sides are set at 90° of rotation apart, hence there are four power impulses during every turn of the drivers, and the torque is a succession of overlapping pulsations.

Since the amount of steam that the boiler can supply is limited, a locomotive can deliver its maximum tractive effort with "full cut-off" only up to the speed where the volumetric demand of the cylinders equals the output of the boiler. At higher speeds the tractive effort would naturally decrease, but to gain efficiency the "cut-off" is shifted so that steam at boiler pressure is admitted during only part of the stroke and expands for the rest of the stroke. Diagram C shows the theoretical tractive effort and horsepower curves for a typical steam freight locomotive and also the drawbar pull and drawbar horsepower obtained on a test run with a dynamometer car. Theoretically, the differences between the two sets of curves should be due to engine resistance, but several factors, such as the test train not being heavy enough to require the full power of the engine and the manner in which the engine was manipulated, would cause the differences shown.

Since some of the moving parts are rotary while others are reciprocating, it is practically impossible to "balance" the moving parts completely at all speeds. Consequently, steam locomotives may be

rough riding, weave from side to side, and "pound the rail" severely, causing damage to both track and engine.

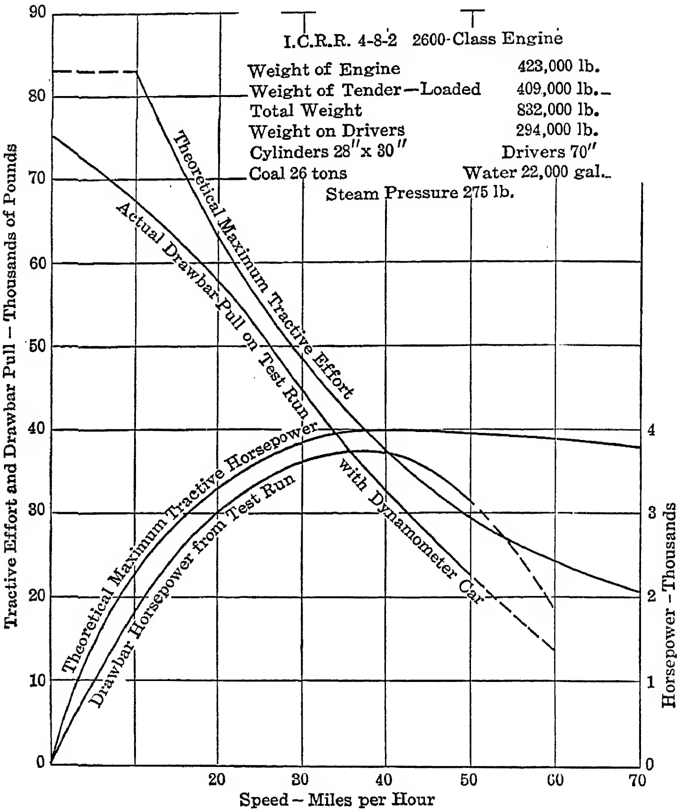


Diagram C.—Steam Locomotive—Tractive Effort, Drawbar Pull, and Horsepower. Data from the Illinois Central Railroad.

Steam locomotives also require a large amount of standby time for getting up steam, taking on fuel and water, cleaning fires, etc., which greatly reduces the time they can be in actual service and thus decreases their overall efficiency.

Every railroad has its own system of locomotive classification. There is, however, an almost universal practice of indicating "types" by the wheel arrangement. Many types have specific names that are often used, but the standard system is to indicate the arrangement by a series of numbers giving the number of wheels in each group. Thus, a "Mountain" type with four leading wheels, eight drivers, and two trailers would be a 4–8–2, while a switcher with eight drivers only would be a 0–8–0.

Articulated engines have two sets of drivers, each with its own cylinders, which are hinged together to give flexibility on curves. Originally these were known as *Mallet* compounds, from the name of the inventor, with high-pressure cylinders for the rear set of drivers and low-pressure cylinders for the forward set. Compounding has practically disappeared, so that the articulates are now simple engines with all high-pressure cylinders. An articulate with two leading wheels, two sets of drivers with eight wheels in each, and four trailers would be a 2–8–8–4.

87. Electric Locomotives. Electric traction has been used for many years for street railways, interurban lines, subways, etc. In all these the cars themselves carried the driving motors. In many cases the cars ran as single units or with one or two trailers. When longer trains were made up, several motor cars were included. Separate locomotives for pulling trains were first used in tunnels, passenger terminals, and other locations of limited extent. Since about 1915 several extensive main line electrifications have been made.

The principal deterrent to electrification is the enormous cost of the construction and maintenance of an adequate system for distributing the power to locomotives at any point on the line. Usually an overhead trolley system is employed, but sometimes a third-rail system is used. The return circuit is through the rails, which requires them to be electrically bonded together, complicating the track circuits for signaling purposes.

Direct current of 500 to 3000 volts is most generally used, although there are a few notable alternating current installations. Regenerative braking is often used. By means of suitable controls, the motors, driven by the momentum of the train, act as generators feeding current back to the line, and thus retard the movement of the train.

Electric locomotives have a number of desirable characteristics. No tender is required, so that this dead weight is eliminated. The tractive effort is the highest at starting, and by suitable controls can be maintained at a high value over a considerable speed range, thus providing smooth starting and rapid acceleration. The torque is smooth, without pulsations; hence the locomotives can be designed with high coefficients of friction, which give more power for the same weight. Since all moving parts are rotary, they can be fully balanced, making the engine smooth riding and easy on the track. They require very little standby time and therefore can be in active service a much greater part of the time than can the steam locomotive.

88. Diesel-Electric Locomotives. The Diesel-electric locomotive is essentially an electric locomotive which carries its power plant along with it. Consequently the expensive power distribution system is not required, and the engines can operate over any line.

The operating characteristics of the Diesel are essentially the same as those of the regular electric locomotive. The smooth starting, rapid acceleration, and capabilities for high speed, when properly geared, combined with low fuel costs and little standby time, have made the Diesel-electric engine very popular for passenger service. Today many "Diesel Streamliners" are covering long distances on very fast schedules.

The Diesel-electric locomotive, when properly geared, is well adapted to heavy freight and switching service, and its use is growing rapidly. In 1947 the orders by American railroads for new freight and switching engines were almost exclusively for Diesel-electrics.

The locomotives are built in "units," and high-power engines are made by coupling several units together. Some units have cabs with a full set of controls, while others do not have cabs and are to be coupled to cab units. Each unit carries one or two Diesel-type oil engines direct-connected to direct current generators supplying current to the driving motors. The controls are so arranged that as more current is fed to the motors the throttles of the oil engines are automatically opened to supply the increased power demands. At present most units for road engines are rated at 2000 horsepower for passenger service and 1800 horsepower for freight service, although larger units are being developed. Switchers are built in several sizes.

Diagram D shows the tractive effort curves of several Diesel-electric locomotives. Those for regular electric locomotives are essentially the same.

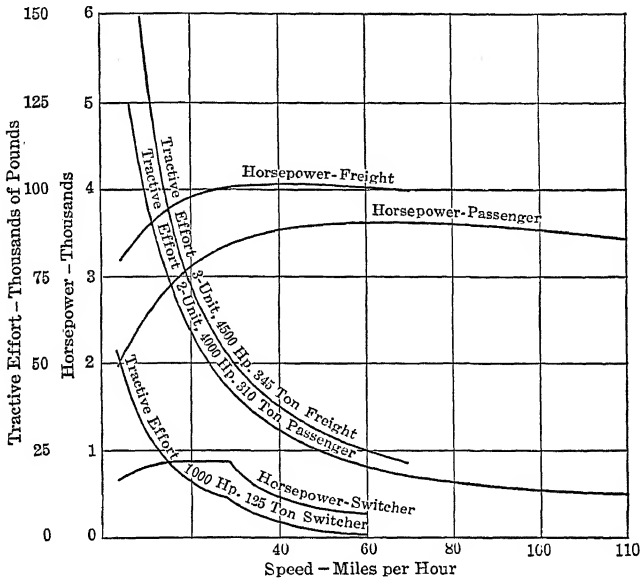


DIAGRAM D.—Diesel-Electric Locomotives—Tractive Effort and Horsepower. Data from Electromotive Division, General Motors Corporation.

89. Weight of Train. The practical application of the foregoing discussion on the relation of grade, tractive, and curve resistance to locomotive performance is to determine the weight of train which a given type of locomotive can haul up a given grade at a given speed.

Since the engine must move itself against grade, tractive, and rolling resistance, the net effort available to move the train, called the *drawbar pull,* is equal to the tractive effort of the engine minus the total resistance of the engine and tender. The drawbar pull, divided by the total unit resistance of the train, will give the weight of train. This may be reduced to the form of an equation as follows:

$$W \quad \frac{T - (R_e + 20G + C)w}{R_t + 20G + C} \tag{1}$$

in which W = weight of train in tons, exclusive of engine and tender.
w = weight of engine and tender in tons.
T = tractive effort of engine in pounds at the given speed.
R_e = tractive resistance of engine and tender in pounds per ton at the given speed.
R_t = tractive resistance of given type of train in pounds per ton at the given speed.
C = curve resistance in pounds per ton, taken as the same for both engine and train.
G = grade in per cent.

If it is desired to find the grade up which a given locomotive can haul a train of given weight at a given speed, Eq. 1 can be changed to the form,

$$G = \frac{T - [R_e w + R_t W + C(W + w)]}{20(W + w)} \qquad (2)$$

Sometimes it is desired to know the tractive effort required to haul a given train up a given grade at a given speed. Since the type, and hence the weight, of engine is unknown, the first step is to determine the drawbar pull which will be required. This can be obtained from Eq. 1, since the numerator of the right-hand member of that equation is the drawbar pull. From this value an engine may be selected from those available, or this requirement may be made the basis of the design of a locomotive with sufficient tractive effort to supply the required drawbar pull under the given conditions.

Illustrative Examples. *Problem 1.* What would be the theoretical weight of a train of 70-ton cars that could be hauled up a straight 0.80% grade at 20 miles per hour, using data from Diagrams A, B, and C?

Taking the required quantities from the diagrams and substituting in Eq. 1,

$$W = \frac{63{,}200 - (5.3 + 20 \times 0.80 + 0)416}{3.6 + 20 \times 0.80 + 0} = 2772 \text{ tons}$$

Problem 2. With a gross daily traffic in each direction of 33,250 tons and for the conditions in Problem 1, (*a*) how many trains would be required daily in each direction, and (*b*) what would be the maximum permissible grade if the number of trains per day were to be one less than from (*a*)?

(*a*) From Problem 1, the number of trains per day would be $33{,}250 \div 2772 = 12$ in each direction.

(*b*) If the number of trains were reduced to 11, the weight of each train would be $33{,}250 \div 11 = 3023$ tons. Substituting in Eq. 2,

$$G = \frac{63{,}200 - (5.3 \times 416 + 3.6 \times 3023 + 0)}{20(416 + 3023)} = 0.73\%$$

Therefore, if the grade could be reduced from 0.80% to 0.73%, the traffic could be handled with one less train per day in each direction.

90. Economics of Grade Reduction. From the preceding discussion it is evident that grades have an important effect on the cost of operation, a fact worth emphasizing. Referring to Problem 1 on page 44, if the grade had been *level* the weight of the train would have been 16,944 tons. With a grade of only 0.10% the tonnage would have been reduced to 10,564 and to 4971 by a 0.40% grade, while the problem shows 2772 tons with an 0.80% grade. These tonnages are equivalent to about 242, 151, 71, and 40 70-ton cars, respectively.

Speed is also a factor since satisfactory train schedules must be maintained. In the preceding paragraph the assumed speed was 20 miles per hour. If the schedules permitted a speed of 10 miles per hour, the full tractive effort of the locomotive could be used, resulting in fewer but heavier trains. Resolving Eq. 1 on this basis, the tonnage for the four grades considered above would be about 26,129, 15,620, 6998, and 3892, respectively, equivalent to about 373, 225, 100, and 56 70-ton cars.

A major problem of the locating engineer is to estimate the saving in the cost of operation by reductions in grades. In Problem 2 on page 44 it was found that one train in each direction could be saved by reducing an 0.80% grade to 0.73%. Assuming the engine district to be 100 miles in length and the *average* total cost of one train-mile as $3.50, the average cost of operating one train over the district would be $350. The elimination of one train, however, would not result in a saving of $350 since overhead expenses would not be reduced, and the actual cost of operating the other trains would be slightly increased because of greater weight.

Considering all factors, railroad economists have estimated the saving to be about 40%. The daily saving, therefore, would be

$\$350 \times 0.40 = \$140$ for one train. For one train each way for 365 days, the saving would then be $\$140 \times 2 \times 365 = \$102{,}200$. This annual saving capitalized at 5% amounts to \$2,044,000; and this is the maximum amount which could be justifiably spent in grade reduction. If it is desired to pay off this indebtedness in a series of years, then the amount which can be expended is the present value of an annuity equal to the annual saving at the given rate of interest for the desired term of years. This is given by the equation,

$$P = S\left(\frac{1-(1+r)^{-N}}{r}\right) \tag{3}$$

in which P is the present value of the annuity, S, for N years and r is the current rate of interest.

91. Minor Grades. Minor grades are those which do not limit the weight of train which can be hauled over the line by one locomotive. Such grades are of three kinds, according to their effect on the performance of the locomotive and on the consequent cost of operation.

First, there are those grades whose drop (vertical height) is sufficiently small that a locomotive can operate them without shutting off steam, or, in other words, the locomotive will not reach a dangerous speed due to the down-grade while exerting a continuous pull on the train. The effect of these grades on train operation is negligible, both as regards the effort required of the locomotive and the time required for the train to traverse a given distance, since the kinetic energy acquired in descending one grade is utilized in ascending the opposite grade.

The second kind of minor grades consists of those whose drop is so great that steam must be shut off to prevent the train from acquiring a dangerous velocity, and hence the effort of the locomotive must be increased on the opposite up-grade. Therefore, there is a loss of power, since the locomotive is working intermittently and not at its greatest efficiency.

The third kind of minor grades consists of those which require the use of brakes in descending them. This causes an enormous loss of energy because part of the kinetic energy gained by the descent is absorbed by the brakes, and hence is not available on the next up-grade. Furthermore, the locomotive is working at a disadvantage, and there is considerable wear on equipment due to the action of the brakes.

92. Ruling Grades. Ruling grades are those which limit the weight of the train which can be hauled by one locomotive. Cars can not be picked up or dropped off along the line to make up a train in accordance with the grades met with, and hence a train must run through from one terminal of a division to the other. Therefore, excluding pusher grades, and such short steep grades as can always be operated by momentum, the maximum grade is the ruling grade on that division.

93. Momentum Grades. Grades which are steeper than the ruling grade, but which are so short that all of the momentum acquired on the preceding down-grade is not dissipated before the top of the grade is reached, are called momentum grades. Railroad engineers are not in accord as to the advisability of using such grades. Some maintain that momentum grades should not be used, because should a train stop on the grade the locomotive would not have sufficient power to start the train and continue. Others hold the view that momentum grades are justified by experience, since such grades are being operated over daily throughout the country with satisfactory results.

Whether or not a proposed grade, steeper than the ruling grade, can be operated satisfactorily with the aid of momentum depends on the speed acquired at the foot of the grade, the minimum desirable speed at the top of the grade, and the length of the grade.

A train moving at a velocity of v feet per second has stored up enough kinetic energy to carry it up a grade whose total height in feet is $h = 1.05 \frac{v^2}{2g}$, if friction is ignored. The equation, $h = \frac{v^2}{2g}$, is the familiar equation for velocity head. Since the rotating wheels have stored up an additional 5% of energy, the coefficient 1.05 is used for determining the velocity head of a moving train. Changing the velocity from feet per second, v, to speed, V, in miles per hour, the velocity head becomes equal to $0.0351 V^2$. Thus if a train has a velocity of 30 miles an hour, and the locomotive is exerting just enough pull to overcome friction, it has sufficient kinetic energy to carry it through a vertical height of 31.6 ft. before it will come to a stop. If a minimum speed of V_t miles an hour is desired at the top of the grade, the total rise of the grade must not exceed $h = 0.0351(V^2_b - V^2_t)$, where V_b and V_t are the speeds at the bottom and at the top of the grade, respectively. The locomotive, however, will normally be exerting a tractive effort greater than that

necessary merely to overcome friction. This effort will vary with the speed. The train resistance also varies somewhat with the speed.

It is evident that the length of the proposed grade (steeper than the ruling grade) is the essential factor in determining whether or not the grade can be operated with the aid of momentum. This length in feet can be computed approximately from the following equation:

$$\frac{71(V^2_b - V^2_t)}{R + 20G - \dfrac{T_a}{(W + w)}} \tag{4}$$

in which V_b and V_t are the speeds in miles per hour at the bottom and top of the grade, respectively; T_a is the average tractive effort of the engine, which may be taken as that at the average speed; R is the average tractive resistance of engine and train, which may be taken as that of the train for the average speed and car weight; and the other symbols are as before. For example, let $V_b = 50$, $V_t = 20$, $W + w = 3200$ tons, $G = 0.80\%$, and the average car weight = 60 tons. The average speed will be 35 miles per hour. From Diagram A, $R = 5.5$. From Diagram C, $T_a = 43{,}000$. Substituting in Eq. 2,

$$L = \frac{71(50^2 - 20^2)}{5.5 + 20 \times 0.80 - \dfrac{43{,}000}{3200}} = 18{,}400 \text{ ft.} = 3.48 \text{ miles}$$

94. Virtual Grades and Virtual Profiles. A virtual grade is the effective value of an actual grade when operated partly by momentum. A virtual profile is one on which the virtual grades are platted instead of the actual grades. The characteristics of a virtual profile are:

1. When a train is standing, the actual profile and the virtual profile coincide.

2. When a train is accelerating, the virtual profile diverges upward from the actual profile.

3. When a train is decelerating, the two profiles approach each other.

4. When a train is moving at a uniform speed, the two profiles are parallel, and the vertical distance between them in feet, to scale, is the velocity head corresponding to the speed.

5. The vertical distance between the two profiles at any point is the velocity head corresponding to the speed at that point.

95. Helper Grades. A helper grade is one which is so steep that one or more extra engines are required to haul the train which one engine can handle on the remainder of the division. It will nearly always happen that some grades on a division will be considerably greater than the majority of the grades, and unless these can be reduced to the general average at a reasonable cost, or a helper engine is used, they limit the weight of train over the division and become the ruling grades.

A helper grade adds enormously to the cost of operation, since the auxiliary engines must be maintained, and, furthermore, they pass twice over the line for each train hauled and therefore do not operate at maximum efficiency. Obviously, therefore, a helper grade must be considerably in excess of the ruling grade on the remainder of the division before its extra cost of operation would be justified.

96. Minimum Grades. The minimum grade which should be used depends upon the drainage of the roadbed. Across fills, there is no objection to a level grade, but through cuts there must be sufficient grade to enable the water which falls or runs into the cut to drain away quickly. If the cut is short, the roadbed grade may be level, and drainage effected by constructing the side ditches on a grade. For long cuts, however, sufficient fall can not be obtained for the ditches unless the roadbed itself is on a grade. Many railroads use a minimum grade through cuts of 0.10%, and this seems to be satisfactory.

97. Choice of Grades. The first step in projecting a grade line on a profile is to determine approximately the ruling grade. The lowest value of the ruling grade is that of a uniform slope between terminals. The maximum value depends on the type and weight of locomotives, the weight of trains, etc., and can never be exactly determined. The general practice is simply to choose a maximum ruling grade which will probably fit the territory through which the line runs. In assuming this maximum value, due consideration should be given to the direction of heaviest traffic, and easier grades secured if possible for trains in this direction.

A grade line is then laid out on the profile, keeping below this maximum, if possible, in such a way that the fills will balance the cuts and the total amount of earthwork will be kept as low as possible in order to reduce the cost of construction. The determination of the most economical grade line will require a number of trials, and each trial should be carefully studied as to its effect on opera-

tion as well as on first cost. After such a line has been laid out, it may be found that the assumed maximum grade has been exceeded at some point, and study must be given as to whether it is possible to reduce this grade, or whether it is justifiable perhaps to increase it sufficiently to be operated as a helper grade. Or it may be found that only one grade approaches the chosen maximum, and again the line must be studied to determine whether it is possible economically to reduce this grade to the next lower. For example, the maximum grade on a division is found to be 1.0%, and there are no others over 0.8%, and possibly only two of these, the next lower being 0.5%. The problem then is to decide the advisability of reducing the 1.0% grade to 0.8%, and then possibly reducing the three 0.8% grades to 0.5%. This process of reducing the ruling grade should be continued until the additional cost of construction equals the sum which can profitably be spent for the purpose of reducing the ruling grade. In these studies consideration must be given to the question of suitable operating speeds in order that a satisfactory schedule can be maintained over the division.

On a given alinement it may be found impossible to reduce the grade to a suitable value at a reasonable cost. In this case it will be desirable to consider a change in the alinement, perhaps to the extent of adding distance to secure a flatter grade. The increased cost of the longer line and the added cost of operating the additional distance must be balanced against the decreased operating costs of the flatter grade.

Highway and Street Grades

In a general way the effect of grades on highway traffic is the same as on railroad traffic. Some differences, however, are found, due principally to (1) the difference in type of motive power and the fact that each vehicle has its own power plant, (2) the difference in the coefficient of friction between the wheels and the roadway, (3) the difference in tractive resistance on the roadway, (4) the difference in the character and cross-section of the roadway, and (5) to a less extent by the fact that the automobile is steered by the driver instead of by rails.

98. Minimum Grades. Where the cross-section is such that surface water can be drained off laterally, the grade may be level for an indefinite distance, as on long embankments. Where the water is collected by side ditches, the grade may also be level, but for a limited distance, depending on how much the ditch can be deepened in order to give it sufficient grade. Where the surface water is col-

lected by gutters they must have adequate slope. When the pavement grade is sufficient for good drainage the gutters are given the same slope. When the street grade is too flat to drain well inlets are placed at short intervals and the gutters sloped each way from them on suitable grades. This results in variable height of curb since the top of curb should follow the general street grade.

Concrete gutters can be satisfactorily built on a minimum grade of about 0.2%. Other types of gutters should have about 0.3%. Open side ditches may have minimum grades of about 0.1%, provided the width can be made adequate.

99. Maximum Grades. Each motor vehicle contains its own power plant, and by means of the gear shift the tractive effort can be greatly increased at the sacrifice of speed. It is possible, therefore, for the normal motor vehicle to climb grades greatly in excess of those desirable for use. Passenger cars on dry roads can negotiate grades up to about 25%, but loaded trucks are limited to about 15%.

Due to the fact that road surfaces may become slippery in bad weather, and to the difficulty of controlling the vehicle on descents, the conditions of safe descent, rather than the ability to climb, fix the maximum safe grades. On main highways where the traffic volume is large and the movement is fast, the maximum grade should not exceed 3%. On less important roads where there are fewer vehicles and speeds can be reduced, grades may be increased up to about 6%.

Streets are sometimes laid out on grades so steep that they are little used by vehicles. Such streets may be paved but esthetic and sanitary benefits rather than convenience to traffic are the justification for the improvement.

100. Tractive Resistance. Owing to the character of the road surface and of the vehicle tires, the tractive resistance on highways is considerably higher than on railroads. It varies also with the kind and condition of the road surface, and with the characteristics of the vehicles, and to a less extent with speed. Since all types of vehicles use the roads, only broad averages of the tractive resistance need be considered, and the effect of speed can be neglected. Table 1 gives ordinary values of the tractive resistance.

101. Grade Resistance. Grade resistance on highways is the same as on railroads, 20 lb. per ton for each per cent of grade.

102. Minor Grades. Minor grades are those which do not affect the normal movement of traffic. Satisfactory running speeds are

TABLE 1

AVERAGE TRACTIVE RESISTANCE FOR MOTOR VEHICLES AT AVERAGE SPEEDS ON DIFFERENT ROAD SURFACES

Kind of Surface	Tractive Resistances Pounds per Ton		
	Poor condition	Good condition	Average condition
Untreated Earth	150–300	70	90
Oiled Earth	100	50	70
Gravel and Macadam	100	40	60
Bituminous Surfaces	75	35	45
Brick and Concrete	75	30	40

obtained at summits, excess speeds are avoided on descents, and brakes are not used to control the speed. Owing to the characteristics of the gasoline motor, such roadways may actually require less fuel than level grades, but the difference is not great and the fact does not justify the designing of undulating grades in place of level grades.

103. Momentum Grades. Momentum grades are those on which the momentum of the vehicle is used to aid in the ascent. This in fact occurs on all minor grades, but may be specifically considered on steeper grades. The maximum economic grade for motor vehicles is that which will permit them to ascend without shifting gears and within a reasonable speed range.

From mechanics it can be shown that,

$$G_a = \frac{3.51(V^2{}_b - V^2{}_t)}{L} + \frac{T - R}{20} \tag{5}$$

where G_a = the ascending grade in per cent.

L = the length of the grade in feet.

V_b and V_t = the speeds at the bottom and top of the grade, respectively.

T = the average tractive effort at the drive wheels in pounds per ton.

R = the tractive resistance in pounds per ton.

The constant 3.51 makes an allowance of about 5% for the inertia of the rotating parts of the vehicle in addition to the inertia of direct movement.

By assuming permissible values of V_b and V_t and of T and R, the permissible grade of a given length, or the maximum length of a given grade, can be determined. T will usually lie between 175 and 200, and values of R can be taken from Table 1.

If the grade is long, the effect of momentum is practically lost before the top is reached; or, if the vehicle is started on the grade, momentum does not act and Eq. 5 reduces to

$$G_a = \frac{T - R}{20} \tag{6}$$

This equation should be used for all grades more than about 2000 ft. long, and also where conditions do not permit the utilization of momentum.

104. Safe Descending Grades. The safe descending grade for economical operation is that which does not require the use of brakes and on which the vehicle will not reach unsafe speeds. It is given by the equation,

$$G_d = \frac{3.51}{L}(V^2_b - V^2_t) + \frac{R}{20} \tag{7}$$

in which G_d is the descending grade in per cent and the other terms are as in Eq. 5.

If the relation between G and L is greater than given by Eq. 7, brakes must be applied on the descent, and this is an economic loss.

105. Limiting Grade. Ruling grades as defined for railroads do not exist on highways, due to the characteristics of the vehicles. If, however, a grade becomes so long or so steep that it limits the number of vehicles which can traverse the road in a given time, it is analogous to the ruling grade, but more properly should be termed the limiting grade. Helper grades, of course, do not exist on highways.

106. Establishing a Grade Line. On a rural road the laying of a grade line is exactly similar to the same process on a railroad, taking into consideration the limiting values. Tangents should be first projected and then connected with vertical curves. The curves should be checked to see that they provide ample sight distance. The line should then be reviewed and modified to give the best

grades and vertical curves and minimum earthwork. Long, easy vertical curves should be the rule, and the appearance should be carefully considered. Short adverse grades on a general ascent should be avoided. The desire to save a few yards of earthwork should not be permitted to introduce undesirable features into the grade line.

Street grades are established in much the same way, but the limiting factors of cross streets, existing improvements along the street, drainage, etc., are often more exacting. Adequate drainage and good appearance are essential factors, whereas the amount of earthwork is a negligible consideration.

Canal Grades

107. Grades. Drainage channels must have sufficient grade to carry the required amount of water and at the same time must not be steep enough to cause excessive velocities and the resulting erosion of the banks. These factors are limited by the character of the soil and by the amount of water to be carried, and are worked out from the hydraulics of open channels.

Transportation canals usually are required to use as little water as possible, and difference of elevation is secured by means of locks. The lock sites must be selected so as to give practically level grades between them.

Pipe-line Grades

108. Grades. Pipe lines for oil and water normally flow full and for a given size of pipe and quantity of flow, the velocity is constant. The resistance to flow is, therefore, that of pipe friction plus the difference in elevation of the two ends, since intermediate rise and fall is self-compensating. The length and steepness of pipe-line grades are, therefore, limited by the mechanical features of installing and maintaining the pipe. Water mains must be buried deep enough to prevent freezing, and this must be considered in establishing the grade line.

Sewer Grades

109. Grades. Sewers are essentially pipe lines operated by gravity. They are designed for maximum capacity when flowing full and for adequate velocity when the flow is the expected minimum. The grade line is, therefore, established with grades between the limits set by the hydraulic characteristics of sewers, and so as to provide the required amount of cover over the pipe.

CHAPTER 4

CIRCULAR CURVES

ALL route alinement consists of

1. Tangents, or straight sections, and
2. Curves, which unite the tangents.

Curves are of three kinds: (1) arcs of circles, (2) arcs of spirals, and (3) arcs of parabolas. The circle is employed for the body of horizontal curves since it is more easily located with the transit and tape than any other curve and also provides uniform running conditions throughout its length. Spirals, also called transition or easement curves, are often used at the ends of circular curves and between the branches of compound curves. Parabolas are used for vertical curves on grade lines and pavement crowns and very occasionally for horizontal curves. Circular curves are classified as *simple*, *compound*, or *reversed*, with or without spirals.

Simple Curves

110. FUNCTIONS.

A simple curve is an arc of a circle which unites two tangents differing in direction. The functions of a simple curve are shown in Fig. 2, and are defined in the following paragraphs:

Point of Intersection—*P.I.*—is the point where the two tangents intersect.

Tangent to Curve—*T.C.*—is the end of the tangent and the beginning of the curve. This point is also termed the Point of Curve, or *P.C.*

Curve to Tangent—*C.T.*—is the end of the curve and the beginning of the tangent. This point is also termed the Point of Tangent, or *P.T.*

Intersection Angle—*I*—is the *deflection angle* between the two tangents, and is equal to the angle at the center from the *T.C.* to the *C.T.*

The Radius of the curve is denoted by *R*.

Tangent Distance—T—is the distance from the $T.C.$ or the $C.T.$ to the $P.I.$ From Fig. 2,

$$T = R \tan \tfrac{1}{2}I \tag{8}$$

Long Chord—$L.C.$—is the chord from the $T.C.$ to the $C.T.$ From Fig. 2,

$$L.C. = 2R \sin \tfrac{1}{2}I \tag{9}$$

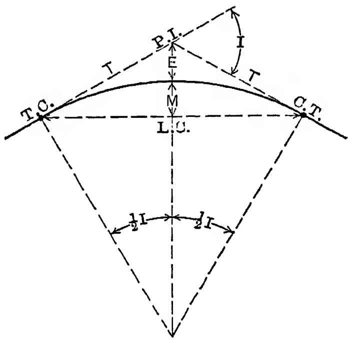


Fig. 2.

Middle Ordinate—M—is the ordinate to the curve from the middle of the Long Chord. From Fig. 2,

$$M = R - R \cos \tfrac{1}{2}I = R \operatorname{vers} \tfrac{1}{2}I \tag{10}$$

External Distance—E—is the distance from the middle of the curve to the $P.I.$ From Fig. 2,

$$E = \frac{R}{\cos \tfrac{1}{2}I} - R = R \operatorname{exsec} \tfrac{1}{2}I \tag{11}$$

Degree of Curve—D—is the unit of *curvature* of the circle, or the angular change of direction of the tangent per unit of curve length. By corollary, it is also the angle at the center subtended by the unit curve length. The curvature at any point of curves other than the circle, such as the spiral, may be expressed by D for the osculating circle at the given point.

As here defined, *the degree of curve is the change of direction or the angle at the center for an arc length of 100.0074 ft.*

The reasons for using this *arc* length, which makes the radius of a 1° curve 5730.0000 ft., are given in section 111 together with some discussion of other definitions of D.

Length of Curve—L—is the distance along the curve between $T.C.$ and $C.T.$

$$L \text{ (in stations)} = \frac{I}{D} \qquad (12)$$

Deflection Angle—$\frac{1}{2}D$—From geometry the angle between two chords, or a chord and a tangent, intersecting on the circumference of a circle is measured by one-half the intercepted arc. Therefore, for each station of 100 ft. on the curve, the angle between the chords is $\frac{1}{2}D$ and is called the *deflection angle.*

Subdeflection Angle—d—For a distance of less than 100 ft. on the curve, the angle between the chords is called a *subdeflection angle.*

If C is the distance in feet,

$$d \text{ (in degrees)} = \frac{D}{2} \times \frac{C}{100} \qquad (13)$$

and

$$d \text{ (in minutes)} = 0.3CD \qquad (14)$$

Equations 13 and 14 are exact only for the arc definition of D, but they are sufficiently accurate and are invariably used irrespective of how D is defined.

Total Deflection Angle is the angle at the $T.C.$ between the tangent and a chord to any point on the curve.

Total deflection angles should be computed and recorded to the nearest 0.1 *minute, and should be turned off as accurately as the given transit will permit.*

111. Relation between R and D. If an arc definition of degree of curve is used, an exact relation between R and D can be found as follows:

The circumference of a circle in terms of its radius is $2\pi R$, and in terms of its degree of curve is length of arc times 360 divided by D; hence

$$R = \frac{\text{length of arc} \times 360}{2\pi D}$$

If the length of arc is taken as 100.007 ft.,

$$R = \frac{5730}{D} \qquad (15)$$

If the length of arc is taken as 100 ft.,

$$R = \frac{5729.58}{D} \tag{16}$$

From either of these equations it is evident that R varies inversely as D. Therefore, knowing the radius of a $1°$ curve, the radius of any other degree of curve can be found by simple proportion. By substituting R in terms of D in Eqs. 8 to 12, it is seen that for any given value of I the various functions of a curve are exactly inversely proportional to D. Thus, if the functions are known for any values of I and D, the functions of any other degree of curve for the same value of I can be found by simple proportion. This is the principal advantage of an arc definition of degree of curve, since a table of functions of a $1°$ curve for all values of I can be prepared and used to determine the functions of any degree of curve more quickly than by the use of the equations.

The objection raised to the use of an arc definition of degree of curve is that it is impossible to measure distances along an arc. This is true; distances can be measured with a tape only along a straight line. In order to locate two points a given distance apart along the arc, a chord length of less than this distance must be used. It would be quite easy to deduct a few hundredths of a foot in measuring the chord, in order that the distance along the arc may be the desired distance, but this is never done in practice.

If the length of arc is taken as 100.007 ft., and 100-ft. chords are used in the field for curves up to 3°, two 50-ft. chords for curves between 3° and 7°, and four 25-ft. chords for curves between 7° and 14°, the theoretical error in each chord length is less than the error in measurement, and the theoretical accumulative error for a curve of average length is well within the permissible error in chaining. Column 4 of Table 2 shows these theoretical errors.

The values of the functions of a $1°$ curve in Table 13 are based on the 100.007-ft. arc definition of degree of curve. If it is desired to use the 100-ft. arc definition, all values in this table must be reduced by 0.0074%.

If the 100-ft. chord definition, or the multiple chord definition, is used, values of R can be computed from Eq. 9.

For 100-ft. chords,

$$R = \frac{100}{2 \sin \frac{1}{2}D} = \frac{50}{\sin \frac{1}{2}D} \tag{17}$$

For 50-ft. chords,

$$R = \frac{50}{2 \sin \frac{1}{4}D} - \frac{25}{\sin \frac{1}{4}D} \tag{18}$$

And for 25-ft. chords,

$$R \quad \frac{25}{2 \sin \frac{1}{8}D} - \frac{12.5}{\sin \frac{1}{8}D} \tag{19}$$

For purposes of comparison, Table 2 has been prepared, showing the radii, arc lengths, and chord lengths for the several definitions of degree of curve.

112. Choice of Curve. The principal factors which determine the degree of curve for railways and highways are the topography and the speed of train or automobile. That degree of curve is used which best conforms to the topography, thereby reducing the amount of grading to a minimum, and at the same time enables the traffic to move at the desired speed. Since time is an important factor on main lines of transportation, the curves on such lines are much flatter, i.e., degrees of curve are smaller, than on secondary lines of transportation. In rough, hilly country, topography is the controlling factor, whereas in flat country speed controls, and curves of low degree, around which traffic can operate safely at maximum speed, are used.

Curves up to 10° are found on main-line American railways, but comparatively few are below 1° or above 6°. Degrees of curve as high as 20° are found on many primary highways. Many of these sharper curves are being replaced by much flatter curves, and highway practice is rapidly approaching that of the railways, because motor vehicle speeds are now as high or higher than train speeds.

113. Field Work. The starting point on any route survey is called station 0, the stake 100 ft. ahead is station 1, the next 100-ft. stake is station 2, and so on. Each stake is marked with its number, and the numbering is continued unbroken around the curves. The *T.C.* and the *C.T.* are very unlikely to come at a full station, but will come at some intermediate point. If the *T.C.* of a curve comes 65.4 ft. beyond station 85, its stationing would be 85 + 65.4. The first stake on the curve is at station 86, which is 34.6 ft. from the *T.C.* Thus the first and the last chords of a curve are of odd lengths and the deflection angles for these sub-chords are computed from Eq. 14.

TABLE 2

VALUES OF THE RADIUS AND LENGTH OF ARC FOR DIFFERENT DEFINITIONS OF DEGREE OF CURVE

$D°$	Arc = 100.0074			100-ft. chord		2 50-ft. chords		4 25-ft. chords		100-ft. arc
	R	Arc	Diff.* per 100′	R	Arc	R	Arc	R	Arc	R
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	5730.00	100.0074	−0.002	5729.67	100.002	5729.61	100.001	5729.60	100.000	5729.58
2	2865.00	"	−0.001	2864.93	100.006	2864.84	100.001	2864.80	100.001	2864.79
3	1910.00	"	0.005	1910.08	100.012	1909.91	100.003	1909.87	100.001	1909.86
4	1432.50	"	−0.002	1432.69	100.021	1432.47	100.005	1432.42	100.002	1432.39
5	1146.00	"	0.000	1146.28	100.032	1146.01	100.007	1145.94	100.002	1145.92
6	955.00	"	0.004	955.37	100.046	955.04	100.011	954.96	100.003	954.93
7	818.57	"	0.008	819.02	100.062	818.64	100.015	818.54	100.004	818.51
8	716.25	"	−0.002	716.78	100.081	716.34	100.019	716.23	100.005	716.20
9	636.67	"	−0.001	637.28	100.103	636.78	100.025	636.66	100.006	636.62
10	573.00	"	0.001	573.69	100.127	573.14	100.031	573.00	100.008	572.96
11	520.91	"	0.003	521.67	100.154	521.07	100.038	520.92	100.010	520.87
12	477.50	"	0.004	478.34	100.183	477.68	100.045	477.52	100.011	477.46
13	440.77	"	0.006	441.68	100.215	440.97	100.053	440.79	100.013	440.73
14	409.29	"	0.008	410.28	100.249	409.51	100.062	409.32	100.015	409.26
15	382.00	"	0.011	383.07	100.286	382.24	100.071	382.04	100.018	381.97
16	358.12	"	0.013	359.27	100.326	358.39	100.081	358.17	100.020	358.10
17	337.06	"	0.016	338.27	100.368	337.34	100.092	337.11	100.023	337.03
18	318.33	"	0.019	319.62	100.413	318.61	100.103	318.39	100.026	318.31
19	301.58	"	0.022	302.94	100.460	301.90	100.115	301.64	100.029	301.56
20	286.50	"	0.025	287.94	100.509	286.81	100.128	286.57	100.032	286.48

* Difference per station between actual measurements and the desired arc of 100.007 ft. when 100-ft. chords are used for curves up to 3°, two 50-ft. chords used for curves between 3° and 7°, and four 25-ft. chords used for curves above 7°.

The word station is used also to indicate a 100-ft. distance. For example, the length of a curve may be expressed in stations.

In staking out a curve, successive points are located by a measurement from the preceding point and by a line of sight. The line is not the same for all points, but is determined by the total deflection angle computed for each point. Thus in Fig. 3, point 1 is located by the chord $T.C.$–1 and the total deflection angle A; point 2 by the chord 1–2 and the total deflection angle B; and point 3 by the chord 2–3 and the angle C.

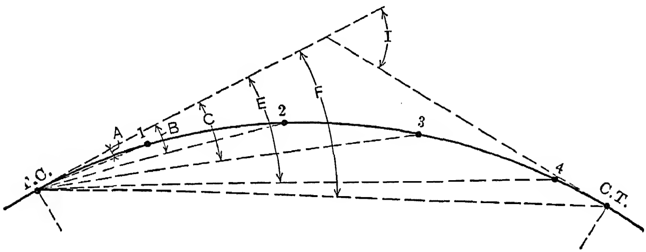


FIG. 3.

The first step in the field work is to determine the station numbers of the $T.C.$ and the $C.T.$ and to compute the total deflection angles for the entire curve. In Fig. 3, $T.C.$—1 and 4—$C.T.$ are odd distances, and 1–2, 2–3, and 3–4 are full stations. Then A is the subdeflection angle (d_1) for the chord $T.C.$–1, and is computed by Eq. 14.

$$A = d_1$$

Then $$B = A + \tfrac{1}{2}D = d_1 + \tfrac{1}{2}D$$

And $$C = B + \tfrac{1}{2}D = d_1 + \tfrac{1}{2}D + \tfrac{1}{2}D$$

$$E = C + \tfrac{1}{2}D = d_1 + \tfrac{1}{2}D + \tfrac{1}{2}D + \tfrac{1}{2}D$$

$$F = E + d_2 = d_1 + \tfrac{1}{2}D + \tfrac{1}{2}D + \tfrac{1}{2}D + d_2 = \tfrac{1}{2}I$$

The total deflection angles are thus computed by successive additions, and the entire series of computations is checked if the last value is $\frac{1}{2}I$. If stakes are placed 50 ft. apart the increments are $\frac{1}{4}D$, and if 25 ft. apart they are $\frac{1}{8}D$, instead of $\frac{1}{2}D$ as above. The form of notes is shown in Fig. 4.

On the ground the $T.C.$ and the $C.T.$ are located by measuring the tangent distance T from the $P.I.$ The curve can then be run in by means of angles and distances as explained above.

TRANSIT NOTES FOR LINE L					
STATION	ALINE-MENT	TOTAL DEFL. ANGLE	CALC. BEAR.	MAG. BEAR.	REMARKS
+ 55.8	⊙ C.T.	35°-41′	N00°-39′W	N00°-40′W	$\frac{1}{2}$ I = 35°41′
			Check		
62		34 -34			
61		32 -34			
10 560		30 -34			D = 4°00′
59		28 -34			I = 71°22′
58		26 -34			
57		24 -34			P.I. 10 555 + 00.3
56		22 -34			T = 1028.7
55		20 -34			
54	⊙	18 -34			
53	4°Curve Right	16 -34			
52		14 -34			
51		12 -34			
10 550		10 -34			
49		8 -34			
48		6 -34			
47		4 -34			
46		2 -34			
45		0 -34			
+ 71.6	⊙ T.C.	0°-00′	N72°-01′W	N72°-10′W	
44					
43					

Fig. 4.

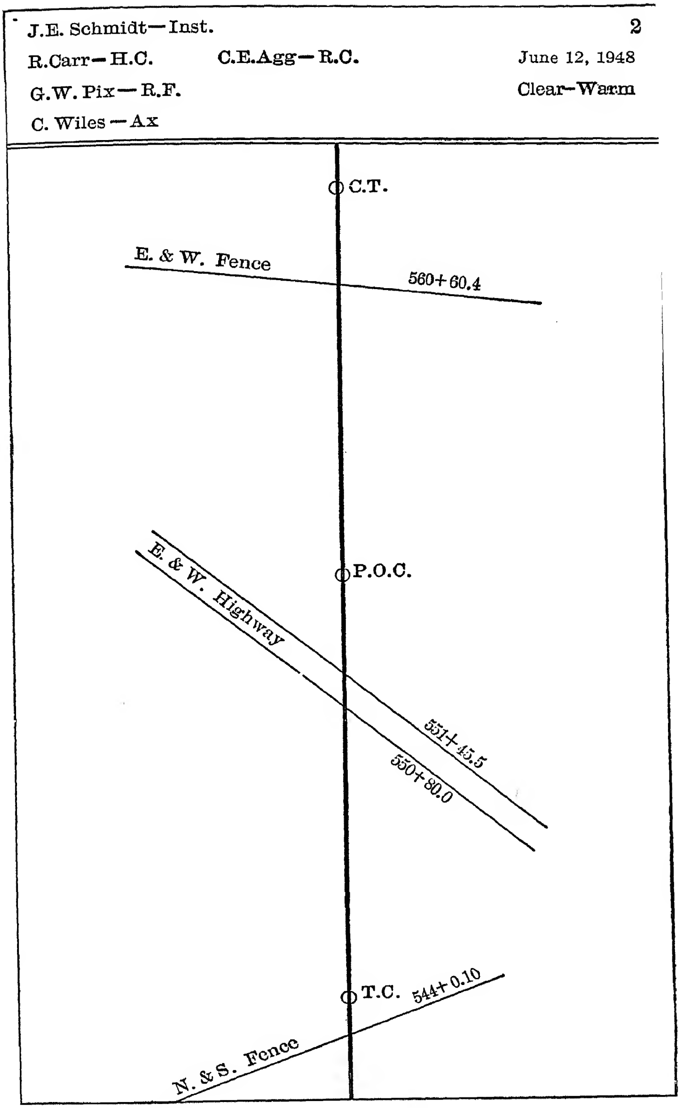

J.E. Schmidt—Inst.
2
R.Carr—H.C.
C.E.Agg—R.C.
June 12, 1948
G.W. Pix—R.F.
Clear—Warm
C. Wiles—Ax
C.T.
E. & W. Fence
560+60.4
P.O.C.
E. & W. Highway
551+45.5
550+80.0
T.C.
544+0.10
N. & S. Fence

FIG. 4.

Since in turning off an angle with a transit, an error as large as 0.25′ may easily be made—which amounts to 0.07 ft. at 1000 ft. from the transit—the length of sight on a curve should never exceed 1000 ft.

It has been found by experience that if the angle between the tape and the line of sight is more than about 30°, the location of the point is inaccurate. Therefore the angle between the line of sight and the tangent at the transit station should never be more than 30°, that is, the product of the length of sight in stations and $\frac{1}{2}D$ should be less than 30°.

It is *good practice* in any case to run a portion of the curve from the *C.T.*, since the errors of surveying can be adjusted more satisfactorily on the curve than at the *C.T.* There should be no more error, however, in either line or distance than is permissible in good chaining under the particular conditions.

If the entire curve can not be run in from the *T.C.* and the *C.T.*, one or more intermediate set-ups on the curve will be necessary. There are, therefore, three possible positions of the transit in running in curves; the *T.C.*, any intermediate point, and the *C.T.* The curve notes as computed above are used in all three cases, as follows:

At the *T.C.* Orient the transit by a sight along the tangent with the plates set at zero. Turn off the total deflection angle of each station, successively, and chain the corresponding distances between them.

At Any Intermediate Point. Orient the transit by a backsight on the *last transit station with the telescope inverted* and the plates set at the total deflection angle of the station sighted at *as recorded in the notes.* To continue the curve, plunge the telescope [1] and set the plates at the total deflection angles of the succeeding stations and measure the corresponding distances between them.

At the *C.T.* Orient the transit by a sight along the tangent towards the *P.I.* with the telescope normal and the plates set at the total deflection angle of the *C.T.*, that is, $\frac{1}{2}I$. To run in the curve, set the plates at the total deflection angle of each station *as given in the notes*, and measure the corresponding distances.

114. The **FUNDAMENTAL PRINCIPLES** are:

When sighting at any station the plates must read the total deflection angle of the station sighted at. When on tangent at any station the

[1] The line of collimation must be in adjustment.

plates must read the total deflection angle of the station over which the transit is set.

If these principles are strictly observed a *mistake* in instrument work is impossible.

Example. The $P.I.$ of two tangents is at Sta. 10555 + 00.3 and $I = 71°22'$. It is desired to connect the tangents with a 4° curve.

$$T = \frac{T \text{ for a } 1° \text{ curve for } I = 71°22'}{D}$$

$$\frac{4114.9 \text{ (from Table 13)}}{4} = 1028.7 \text{ ft.}$$

$$\text{Sta. } T.C. = \text{Sta. } P.I. - T = (10555 + 00.3) \quad (10 + 28.7)$$

$$= 10544 + 71.6$$

$$L = \frac{I}{D} = \frac{71°22'}{4} \quad \frac{71.367}{4} = 17.842 \text{ Stations.}$$

$$\text{Sta. } C.T. = \text{Sta. } T.C. + L = (10544 + 71.6) + (17 + 84.2)$$

$$= 10562 + 55.8$$

The distance from the $T.C.$ to the first Station on the curve is 28.4 ft., hence

$$d_1 = 0.3\,CD = 0.3 \times 28.4 \times 4 = 34'$$

$$\text{Deflection angle} = \tfrac{1}{2}D = 2°00'$$

The distance from the last Station to the $C.T.$ is 55.8 ft., hence

$$d_2 = 0.3 \times 55.8 \times 4 = 67' = 1°07'$$

Total Deflection Angle of	$T.C.$ =	0° 00′	
" " " "	Sta. 10545 =	0° 00′ + 0° 34′ =	0° 34′
" " " "	" 10546 =	0° 34′ + 2° 00′ =	2° 34′
" " " "	" 10547 =	2° 34′ + 2° 00′ =	4° 34′
" " " "	" 10562 =		= 34° 34′
" " " "	$C.T.$	= 34° 34′ + 1° 07′ =	35° 41′
			= $\frac{1}{2}I$ (check)

These results are recorded in the form shown in Fig. 4.

115. Adjusting Field Errors. It generally happens that there are field errors of closure at the end of a curve, or at an intermediate junction point if parts of the curve are run from opposite directions, which are likely to be too large to be neglected even if within the permissible limits of good surveying. Errors in distance can always be discarded, since they will not affect the alinement, and the track or pavement will always be laid continuously. Lateral errors, however, even if small, will form objectionable kinks in the alinement and therefore should always be eliminated by proper field adjustments in setting the center-line stakes.

Since the *C.T.* of an unspiraled curve is on a fixed tangent, the curve must be adjusted to meet it. If the curve is spiraled, the final spiral should always be backed in from the *S.T.* to the *C.S.*, and the *C. S.* thus becomes the fixed point to which the circle must be adjusted.

To illustrate the procedure, it will be assumed that an unspiraled curve with stakes placed at 50-ft. intervals and with a final chord of 30 ft. to the *C.T.* has an outward error of closure of 0.23 ft., and that the error may be adjusted out at the rate of 0.10 ft. per 100 ft. Since the *C.T.* must remain fixed on the tangent, the last point, set opposite it in running the curve, is discarded, which is equivalent to moving the curve point *inward* the required 0.23 ft. The next point back, at a distance of 30 ft., is then moved inward $0.23 - (0.3 \times 0.10)$ or 0.20 ft. The next preceding point, 50 ft. back, would be moved $0.20 - 0.05$ or 0.15 ft. The next two points would be moved 0.10 and 0.05 ft., respectively, which would eliminate the error in a total distance of 230 ft. Theoretically, this will not result in a perfect circle, but it will provide a smooth riding curve well within the permissible limits of accuracy.

If the junction point of the field work is on the circle, as it should be on long curves, the final point at the junction should be placed midway between the two points located from opposite directions. Half of the total error is then run out in each direction in the manner indicated above.

Experience with string-lining (see Chapter 6) indicates that variations of 3 minutes in the degree of curve are permissible in the best work and 6 minutes in average work. One minute of angle will result in a lateral movement of 0.03 ft. at a distance of 100 ft. and is equivalent to change in the degree of curve of 2 minutes. Since 6 minutes change in the degree of curve would mean 3 minutes in the deflection angle, or a lateral displacement of 0.09 ft. per station, it is evident that lateral errors of closure may be run out at any rate up to about 0.10 ft. per 100 ft.

116. Offsets from Chords. Case 1. In addition to his chaining duties the head chainman also sets the stakes on line as directed by the transitman. Obviously this work is much facilitated if the head chainman can quickly approximate the position of the stakes. This can be done by an offset from the line through the last two stations. For example, in Fig. 5, A and B are the last two stations set. Produce AB to E the distance C from B. F is located by the offset $2y$ from E and the distance C from B. y can be found by Eq. 20. If E is carefully lined in and C and $2y$ measured with the tape, the point can be located with considerable accuracy; and, if a *transit is not at hand*, this method can be employed in locating an entire curve. However, this method is most valuable as an aid to

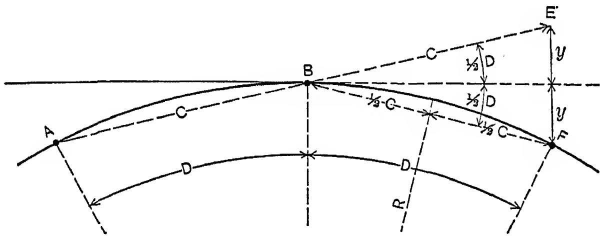


Fig. 5.

the head chainman in approximating the position of the stakes, and it should *always* be used. If E is only roughly lined in and $2y$ is paced, the points can be located within less than one foot, which is sufficient for the purpose.

From Fig. 5,

$$y : C :: \tfrac{1}{2} C : R$$

Whence $$y = \frac{C^2}{2R} \text{ (}C\text{ and }R\text{ expressed in feet)}$$

Expressing R in terms of D, we have

$$y \quad \frac{C^2D}{11460}$$

If n represents C expressed in stations, then

$$y = 0.873n^2D \tag{20}$$

y is the *tangent offset* to the point F.

For $\quad C = 100 \text{ ft.}, n = 1,$

and $\quad y = 0.873D, \quad \text{and} \quad 2y = 1.746D = 1\tfrac{3}{4}D \text{ (approx.)} \qquad (21)$

For $\quad C = 50 \text{ ft.}, n = \tfrac{1}{2},$

and $\quad y = 0.22D, \quad \text{and} \quad 2y = 0.44D$

The method as given above is applicable only when the chords are of uniform length. Normally there is an odd length of chord at the beginning of the curve, and consequently the first two stations on the curve must be located somewhat differently. The first station is located by using a tangent offset y_1, computed from Eq. 20 for the proper chord length c. The offset Y_1 from this first chord, produced the normal chord distance C, to a tangent to the curve at the first station is found from the proportion $Y_1 : y_1 :: C : c$. The offset from this tangent to the second station on the curve is the distance y for the chord C as given above. Hence, the total offset for the second station from the first chord produced is $Y_1 + y$. From the second station the curve is continued as previously explained. At the end of the curve a similar problem arises and is solved in the same way.

Case 2. Occasionally there are conditions under which the deflection angle method of running in curves can not be used to

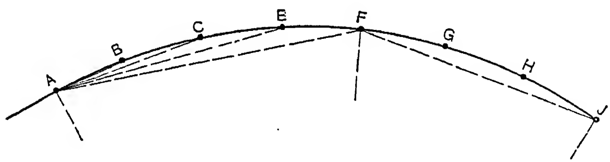


FIG. 6.

advantage. For instance, in heavily wooded country it may not be economical to locate every station by the deflection method on account of the large amount of clearing necessary for the lines of sight. Fig. 6 illustrates this condition. If the stakes B, C, E, and F were all set from A it would necessitate the clearing of a line to each or practically the entire area between the curve and the chord AF. If, on the other hand, F were set from A by its total deflection angle and the chord AF, and B, C, and E located by offsets from this chord, the only clearing required would be along AF

and the short offsets. The transit would then be moved to F and the process repeated to J, and so on.

The chord AF is the $L.C.$ of a curve whose value of I is the central angle of the curve $ABCEF$, and therefore can be determined from Eq. 9 or from Table 13. In chaining this chord, temporary stakes are set at the full station distances, although perpendicular offsets from these will not give the exact location of the curve stations since the chord AF is shorter than the line $ABCEF$. This error may be neglected for the following reasons: (1) in practice the error at any point will rarely exceed one foot; (2) the perpendicularity of the offsets is established by eye, hence is not exact; and (3) the approximate location of these stations is sufficient since this method would be used only on first location to determine the profile. The offsets are computed as follows:

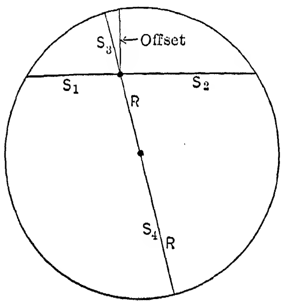


FIG. 7.

From geometry the products of the segments of two intersecting chords are equal. Then from Fig. 7,

$$S_1 \times S_2 = S_3 \times S_4$$

or

$$S_3 = \frac{S_1 \times S_2}{S_4} = \frac{S_1 \times S_2}{2R - S_3}$$

Since S_3 is small compared with $2R$ it may be neglected in the denominator, and since in practice the difference between S_3 and the offset will rarely reach 0.05 ft., we may write

$$\text{Offset} = \frac{S_1 \times S_2}{2R} = \frac{S_1 \times S_2 \times D}{2 \times 5730} \text{ (Approx.)}$$

Reducing S_1 and S_2 to stations, we have

$$\text{Offset (in feet)} = 0.873 S_1 S_2 D \text{ (Slide Rule)} \qquad (22)$$

117. Middle Ordinate Method. If the curve is to be staked out with the tape alone, the middle ordinate method may be more con-

venient than the offsets-from-chords-produced method, because the middle ordinates are shorter than the offsets. As shown in Fig. 8, point a is located by a tangent offset from the chord produced. This offset is $\frac{1}{4}y$, and is equal to $0.218\,n^2D$. If the length of chord used is 100 ft., n is 1 and the offset is $0.218D$. The middle ordinate at point a is m, which is also equal to $\frac{1}{4}y$, or $0.218D$. The middle ordinates at points b, c, d, and e are also equal to $0.218D$. Point b is located by swinging the tape about the $T.C.$ as a pivot, until the 50-ft. point on the tape is the distance m from point a. The tape must be stretched taut and m carefully measured. This operation can be carried out more accurately if a strong, light string, such as a silk fishline, be used instead of the tape. Points c, d, and e are located in the same manner. This method requires an extra man to read the middle ordinates.

The middle ordinate can be obtained trigonometrically. In triangle 1, Fig. 8, it is evident that $m = R \text{ vers } \frac{1}{2}\theta$.

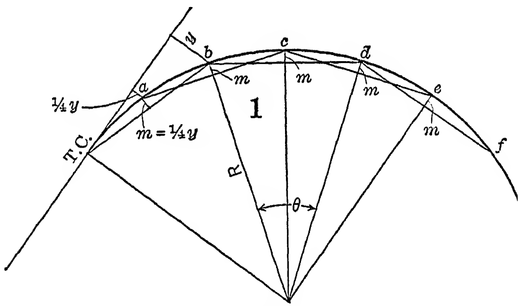


FIG. 8.

If the chord, C, is 62 ft. in length, the middle ordinate m, in inches, is equal approximately to the degree of curve, since

$$m \text{ (inches)} = 12 \times 0.218 \times 0.62^2 \times D = 1.006D \qquad (23)$$

This is the easiest and quickest method of determining the degree of an existing curve.

The middle ordinate for any constant length of chord varies directly as the degree of curve. The sum of all the middle ordinates of a curve varies directly as the central angle of the curve. As long as the central angle remains constant, the sum of the middle

ordinates remains constant, regardless of the degree of curve used to connect the tangents, provided the same length of chord is used. This is true even if the curve is irregular, and certain portions are sharper than other portions.

The middle ordinate method is the basis of the string-lining method of realining existing curves, which is treated in Chapter 6.

118. Curve through a Fixed Point. Sometimes the topographic conditions make it necessary to determine the radius of the curve which will pass through (or near) a fixed point and connect the two given tangents.

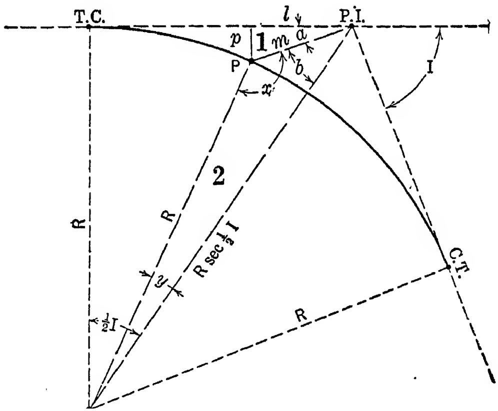


FIG. 9.

In Fig. 9 the intersection angle I and the location of point P are known. The point P is usually known through its coordinates l and p from the initial tangent.

In triangle 1, the altitude p and the base l are known, whence the angle a and the hypotenuse m can be computed. Or a and m can be measured in the field.

In triangle 2, the side m is known and the angle

$$b = 90 - \left(a + \frac{I}{2}\right):$$

By the sine proportion,

$$\sin x = \sin (180 - x) = \frac{R \sec \frac{1}{2}I}{R} \sin b = \sec \tfrac{1}{2}I \sin b = \frac{\sin b}{\cos \frac{1}{2}I}$$

and, $$y = 180 - (b + x)$$

The angles and the side m are now known, whence the radius R can be determined by the sine proportion. D is then obtained from R to the nearest 10′, unless the curve must pass exactly through P.

119. Broken-back Curves. When two curves in the same direction are separated by a short tangent, the layout is termed a broken-back curve. Such curves possess poor riding qualities and their appearance is bad, which is especially objectionable on highways.

The maximum length of tangent which can exist and still make the layout broken-backed is indefinite. If the intervening tangent is less than the sum of the two runoff distances normally placed on the tangents, the curve is undoubtedly broken-backed. Since it is undesirable to run out the superelevation and immediately run it in again, there should be a piece of tangent of indefinite length between the two points where the superelevation becomes zero. It would seem that this should at least equal the sum of the runoff distances. This means that any layout with a tangent less than about 500 ft. between two curves in the same direction should be considered a broken-back curve.

Broken-back curves can usually be avoided with a little care in projecting the location. Spirals can be used to replace all or part of the tangent, or the two curves can be compounded with a third. Spirals are considered in Chapter 5. Compounding can be used both on new work and to improve existing curves. The most common case of compounding is when the two degrees of curve and the length of the intervening tangent are known, and it is desired to compound with a chosen curve and find the position of the two points of compounding.

In Fig. 10, D_1, D_2, and l are known and D is chosen.

In triangle 1, the base l and the altitude $R_2 - R_1$ are known, whence the angles a and b and the hypotenuse O_1O_2 can be computed.

In triangle 2, the three sides are known, whence the three angles c, d, and e can be determined (see appendix).

Angles x and y are now found by arithmetic.

Since the central angle x and the degree of curve D_1 are now known, the distance back from the $C.T.$ to the $C.C._1$ can be readily

found. In the same way the distance from the $T.C.$ to the $C.C._2$ can be determined, and also the length of the D curve.

A small station equation will exist at the $C.C_2$, equal to the difference in distance along the old and the new alinements between the $C.C._1$ and the $C.C._2$.

Sometimes it is desired to know how much the center line will

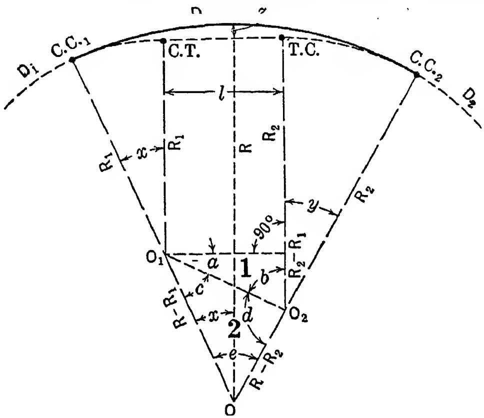


FIG. 10.

be moved laterally by such compounding. From the figure this distance is

$$z = R - R_1 - (R - R_1)\cos x = (R - R_1)\operatorname{vers} x \text{ (approx.)}$$

This equation is exact when $R_1 = R_2$ and is sufficiently accurate unless there is a very large difference between R_1 and R_2.

Change of Location Problems

After the line is located according to the paper projection and a profile made, it may be found that, owing to inaccuracies in the platted topography, a considerable cut or fill comes on a hillside and may be eliminated by shifting a portion of the alinement. Occasionally other considerations than earthwork make such a shift desirable.

120. Problem 1. In Fig. 11, the line as originally located (shown dotted) involves considerable earthwork which could be eliminated by shifting the curve the distance p at or near its middle point.

The problem is to find the new degree of curve and the change in the position of the $T.C.$ and the $C.T.$ p is the difference in the external distances of the two curves, or

$$E_{\text{new curve}} = E_{\text{old curve}} + p$$

The new degree of curve may be computed from the new external by Eq. 11 or by means of Table 13, which is the quicker and more common method. The estimated value of p will usually give an odd value of D which is undesirable; hence the value of D to be used is taken only to the nearest ten minutes. The distance that

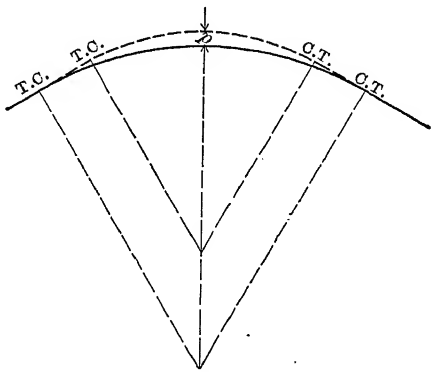


FIG. 11.

the $T.C.$ and the $C.T.$ are shifted is equal to the difference in T for the two curves.

121. Problem 2, Case 1.

In Fig. 12, the line as originally located (shown dotted) involved considerable earthwork between A and B, which could largely be eliminated by shifting the tangent the distance p parallel to its original position, which involves a change in the location of *TWO* curves.

The problem is to find the change in the $T.C.$'s and the $C.T.$'s of the two curves, the degrees of curve remaining unchanged. In triangle 1, the sides m and l can be readily computed since I and p are known. $m = n =$ the required change in the $T.C.$ and the $P.I.$ p will rarely exceed 100 ft.; hence the new tangent is best located

by offsets from the old tangent. The new *C.T.* is then located from the old *C.T.* by rectangular coordinates one of which is p and the other is the side l of triangle 1.

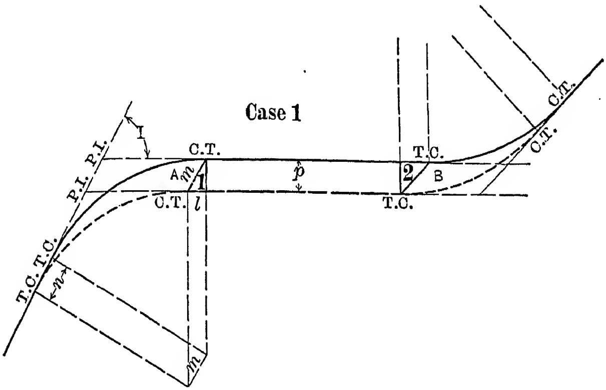


FIG. 12.

The second curve is located similarly using triangle 2.

122. Problem 2, Case 2.

Owing to some special cause, for example a stream-crossing as

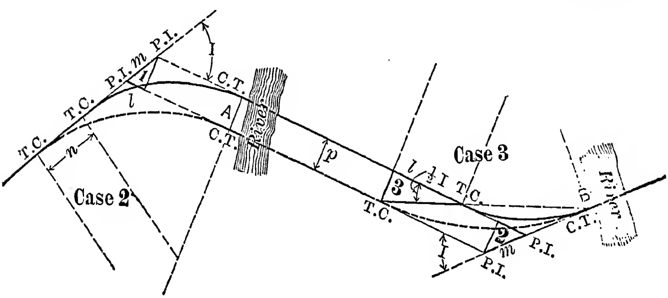


FIG. 13.

shown in Fig. 13, it may be undesirable to move the *C.T.* forward as in Case 1. This condition can usually be met by placing the new *C.T.* *approximately* on the same radial line as the old *C.T.*

This involves a change in the degree of curve and in the position of the $T.C.$

In triangle 1, I and p are known, and l and m are computed.

$$T_{\text{n}} = T_{\text{ol}} - l$$

whence the new D is easily found from Table 13 and, as in Problem 1, is taken only to the nearest 10′. (The approximation in the position of the $C.T.$ is due to this not using the exact value of D.) A new value of T is then found which agrees with the D that is used. Then

$$n = T_{\text{old curve}} + m - T_{\text{new curve}}$$

123. Problem 2, Case 3. This case is similar to Case 2, except that the $C.T.$ is on the fixed tangent instead of the one being changed, hence the solution is somewhat different.

Solve triangle 2 for the side m, then

$$T_{\text{new curve}} = T_{\text{old curve}} - m$$

whence the new degree of curve is obtained from Table 13.

Solve triangle 3 for the side l. Then the new $T.C.$ can be located from the old $T.C.$ by the coordinates l and p.

As in the previous problem, D should be taken only to the nearest 10′. This necessitates a slight change in both the $T.C.$ and the $C.T.$, since T for this value of D is not the same as that used in the above computations. It must be remembered that the $T.C.$ becomes the $C.T.$, or vice versa, when the line is run in the opposite direction.

124. Problem 3. Sometimes curves are run in beginning at the $T.C.$ without previously locating the $P.I.$ and the $C.T.$, and checking the direction of the forward tangent. On reaching the $C.T.$ and projecting the forward tangent, it is found that the tangent does not pass through some controlling point. It is therefore necessary to change the direction of the tangent by shifting the $C.T.$ This is most readily accomplished by measuring the necessary change of direc-

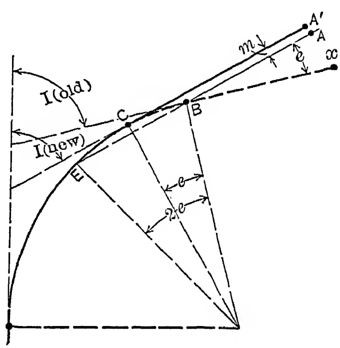


FIG. 14.

tion with the transit, and then changing the length of the curve to correspond. For example, in Fig. 14, the curve is run to B as a $C.T.$ and the tangent is found to pass through x, whereas it is necessary to pass through A. The angle e is measured with the transit. The length of curve corresponding to e is BC, whence the $C.T.$ is moved back to C. The forward tangent then takes the position CA', parallel to BA, and misses A by the distance m, which is the middle ordinate of the curve BE.

$$BE = 2BC = \frac{2e}{D}$$

$$m = \frac{7}{8} \times \frac{BE^2}{4} \times D = 0.875\frac{e^2}{D}$$

As m will rarely reach 5 ft., the line CA' meets the requirements.

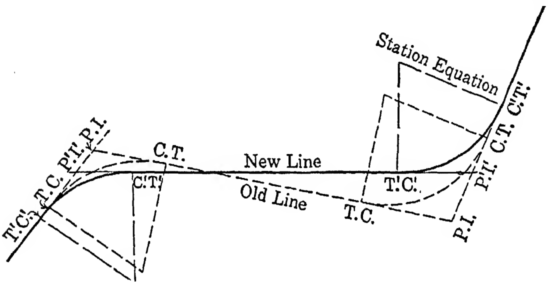


FIG. 15.

125. Problem 4. Occasionally it may be found desirable to change both the position of a tangent and its direction. In this case it is best to treat the problem as one in relocation. The new tangent is run out on the ground, the new $P.I.$'s located and the new values of I determined. The same degrees of curve may be used or others chosen to better fit the topography. The new alinement is then run in without reference to the old except that the distances along both are computed and a station equation made at the $C.T.$ of the second curve. This problem is illustrated in Fig. 15.

Compound Curves

126. Conditions for Use. A compound curve is a combination of two or more simple curves in the same direction with a common tangent at the point of junction. *A compound curve should never be used except under conditions where a simple curve will not meet the requirements.* In rough country it may happen that a large volume of earthwork can be avoided by making one part of a curve sharper than another, resulting in a compound curve.

For example, in Fig. 16, the line is located as a compound curve ABC, requiring little earthwork and keeping on the bank of the stream. If the simple curve AB had been produced to F to end

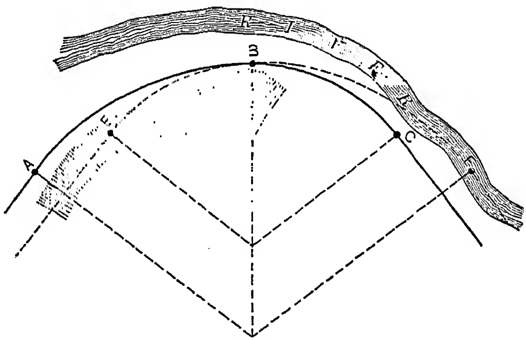


FIG. 16.

in a parallel tangent, it would have fallen in the river. Or if the simple curve CB had been prolonged to E, it would have pierced the cliff.

The degrees of curve and the point of compound curve, $C.C.$, are chosen to fit the contours and other governing conditions, and the central angles of the branches are scaled from the map.

This is one of many cases where a compound curve is applicable.

From the standpoint of operation a compound curve is better than two simple curves separated by a short tangent, hence it should be used in such cases. *In flat country a compound curve is inexcusable on main line.* If the degrees of curve of two adjacent branches of a compound curve differ by more than 2° 00′, an easement curve should be inserted between the branches, and provision

for this should be made in locating the curve. (See Spirals, section 147.)

The nomenclature and positions of the functions are shown in Fig. 17.

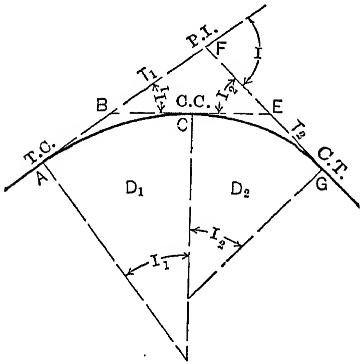


FIG. 17.

127. Problem 1.

In new work I will be measured and I_1, I_2, D_1, and D_2 will be determined from the preliminary maps. It is desired to know the values of T_1 and T_2 in order to set the $T.C.$ and the $C.T.$ and to run in the curve.

In Fig. 17, BE is the common tangent at the $C.C.$

$$BE = BC + CE$$

$BC = AB$ is the tangent distance for a D_1 curve for an intersection angle I_1; and $CE = EG$ is the tangent distance for a D_2 curve for an intersection angle I_2 and both are computed by Table 13.

Then in the triangle BEF, one side and the adjacent angles are known from which the sides BF and EF are computed.

$$T_1 = AB + BF, \quad \text{and} \quad T_2 = EF + EG$$

128. Problem 2. Occasionally the positions of the $T.C.$ and the $C.T.$ are fixed within narrow limits, giving unequal tangent distances, which necessitates a compound curve.

The degree of curve of one branch must be chosen before the problem can be solved. The problem then is to determine the degree of curve of the second branch and the central angles I_1 and I_2.

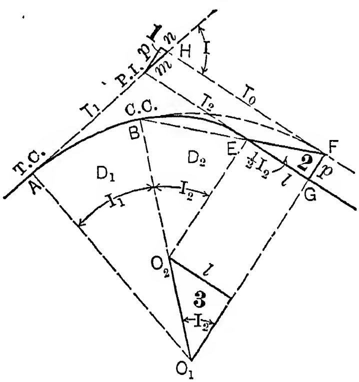


FIG. 18.

In Fig. 18, I, T_1, T_2, and D_1 are known. $AH = HF$ the tangent of D_1 curve for the intersection angle I.

In triangle 1,

$$m = T_0 - T_1$$

and hence the triangle can be solved for p and n.

In triangle 2,

$$l = T_0 + n - T_2$$

Since p is known, the triangle can be solved for the angle at E, which is $\frac{1}{2}I_2$.

$$I_1 = I - I_2$$

In triangle 3, since l and I_2 are known, the side $O_1O_2 = R_1 - R_2$ can be solved for, whence D_2 is determined from Table 10.

129. Field Work. A point on one branch of a compound curve can not be located with the transit set on a point on another branch, since in order to run in a circular curve with the transit and tape, the transit and the points to be located must be on the same circumference. Each branch of a compound curve may therefore be run in independently as a simple curve; but the following method is more convenient.

The total deflection angles for the first branch are computed from the *T.C.* as for a simple curve to agree with D_1 and must check at the *C.C.* by equaling $\frac{1}{2}I_1$. The total deflection angles for the second branch are computed to agree with D_2, but are added on continuously to those of the first branch; hence the check at the *C.T.* is that the total deflection angle must equal $\frac{1}{2}I_1 + \frac{1}{2}I_2 = \frac{1}{2}I$. The transit is oriented at any point in the same manner as on a simple curve by setting off the total deflection of the station sighted at, remembering only that *a sight can not be taken past the C.C.*, since this would be sighting at a point not on the circle through the transit point and the deflection angle method would not apply. The transit may be set at the *C.C.*, however, since this point is common to both circles.

The errors of field work should be accumulated and corrected at some point on the curve other than at the *C.T.* This may be done at the *C.C.* if desired.

Change of Location Problems

130. Problem **1.** This problem arises under conditions similar to those in Problem 2, Case 1, of simple curves, section **121.**

Given, as shown in Fig. 19, a located compound curve ABC. It is desired to move the forward tangent laterally a distance p. The simplest solution of this problem is to retain the same degrees of curve and to shift the $C.C.$ from B to E.

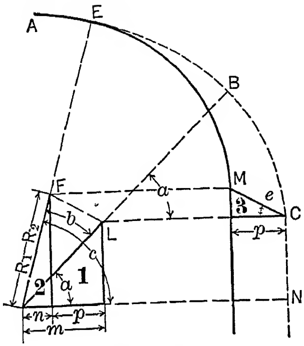


FIG. 19.

$$BE \text{ (in feet)} = \frac{b}{D_1} 100$$

Therefore the problem resolves itself into solving for the angle b.

In triangle 1, since a is known and the hypotenuse is the difference in the two radii, the base m can be computed. Then $n = m - p$.

In triangle 2, the hypotenuse and the base are known, whence the angle c is computed.

$$b = c - a$$

In triangle 3, $\qquad e = 90° - \frac{1}{2}b - a$

whence the coordinates of M, the new $C.T.$, from the old $C.T.$ can be determined.

Three other cases of this problem arise: (1) the same as above, except that the tangent is moved *outward*; (2) the curve of longer radius may fall on the tangent to be moved *inward*; and (3) the curve of longer radius may fall on the tangent to be moved *outward*.

The solution of each of these cases is the same as that given above except that the various points take different relative positions. If the same relative construction lines are drawn and the corresponding points are used, no confusion should arise.

131. Problem 2. This problem arises under conditions similar to those in Problem 2, Case 2, of simple curves, section **122**.

Given, as shown in Fig. 20, a located compound curve ABC. It is desired to move the forward tangent laterally a distance p, and to keep the $C.T.$ approximately on the same radial line, which involves a change in the degree of curve of the second branch and in the position of the $C.C.$

In triangle 1, a is known and the hypotenuse is the difference in the two given radii. Therefore the base m and the altitude n can be computed.

In triangle 2, the base n is now known and the altitude l can be determined since R_1, R_2, p, and m are known. The angle at $F = \frac{1}{2}b$ (Why?) can therefore be determined. Then

$$c = b - a$$

and the $C.C.$ is shifted the distance BE (in feet) $= \dfrac{c}{D_1} 100.$

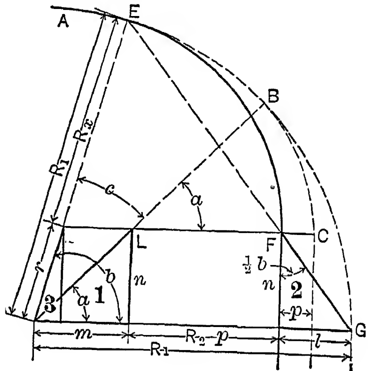


FIG. 20.

In triangle 3, the angle b and the altitude n being known, the hypotenuse r can be computed from which R_x is determined, and the corresponding degree of curve D_x is found from Table 10, and is taken to the nearest 10′. Then since E and b are already fixed, this will change the value of p slightly and will shift the $C.T.$ a short distance along the line EG from F, its theoretical location. The coordinates of this new position are given by the equations

$$x = h \sin b, \quad \text{and} \quad y = x \tan \tfrac{1}{2}b,$$

in which x is the movement along the tangent, y is the change in p, and h is the difference between the computed value of R_x and the value actually used.

Three other cases of this problem arise, depending on whether the longer or the shorter radius curve is on the tangent to be moved, and on whether the tangent moves inward or outward. In all of these cases the solution is relatively the same.

Reversed Curves

132. Conditions for Use. A reversed curve is a combination of two simple curves of opposite curvature with a common tangent at the point of junction.

A reversed curve should never be used on main lines of transport. The shock to trains due to the sudden reversal of curvature on railroads, the difficulty of properly steering motor vehicles on highways or vessels in canals, and the danger of erosion in drainage channels all combine to make reversed curves undesirable for major traffic movements. Furthermore, it is impossible to superelevate properly the outer rail, or outer edge of roadway, at or near the point of reversal, and this is essential to smooth and safe operation of trains or automobiles. If conditions require two curves of contrary curvature close together, they should be separated by sufficient tangent to run out the superelevation of each, or each should be provided with a spiral, in which case the "points of spiral" (*S.T.* and *T.S.*) may be made coincident.

In railroad yards and connections reversed curves may be permissible, since the speed is low. Since practically all such cases occur in connection with turnouts, further consideration of reversed curves on railroads will be given in Chapter 8.

The main alinement of roads and streets should also be free from reversed curves, but frequently such curves are required, or may be desirable, in curb lines and other minor locations.

Two general cases of reversed curves arise as follows:

133. Case 1. In Fig. 21, the two parallel tangents are to be connected by a reversed curve. R_1, R_2, and p are given.

From triangle 1,

$$l_1 = R_1 \sin a, \quad \text{and} \quad m_1 = R_1 \operatorname{vers} a$$

From triangle 2,

$$l_2 = R_2 \sin a, \quad \text{and} \quad m_2 = R_2 \operatorname{vers} a$$

Whence,

$$p = m_1 + m_2 = (R_1 + R_2) \operatorname{vers} a$$

$$\operatorname{vers} a = \frac{p}{R_1 + R_2} \tag{24}$$

And

$$L = l_1 + l_2 = (R_1 + R_2) \sin a \tag{25}$$

From these two equations and the ordinary functions of a simple curve, all ordinary cases of reversed curves between parallel tangents can be solved.

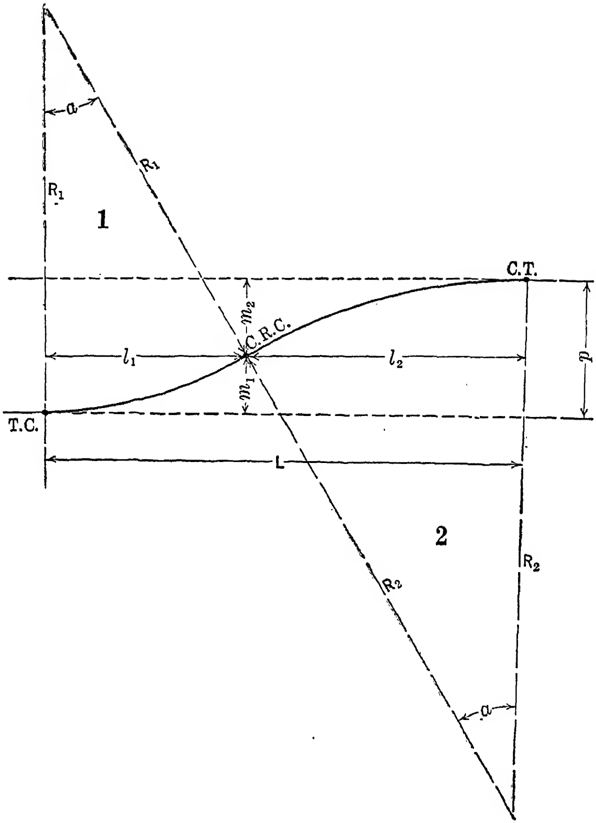


FIG. 21.

134. Case 2. In Fig. 22, the two tangents, intersecting with the angle I, are to be connected by the reversed curve in which T_1, R_1, and R_2 are known, and the tangent distance T_2 and the central angles of the two branches are required.

In triangle 1, the base T_1 and the angles are known, from which the sides l and m can be computed.

In triangle 2, the hypotenuse is $R_1 - m$, and the angles are known, whence the base p and the altitude n are determined.

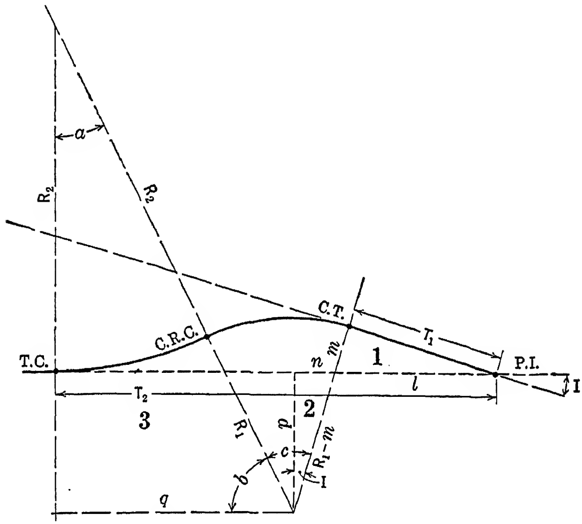


FIG. 22.

In triangle 3, the base is $R_2 + p$, and the hypotenuse is $R_1 + R_2$ whence the angles a and b and the distance q can be found.
Then,

$$c = I + a$$

and

$$T_2 = l + n + q$$

Special Highway Problems

135. Widening Pavements on Curves. Observation has shown that the rear axle of a motor vehicle remains essentially radial on curves. Since the chassis frame is rigid, the front wheels must travel on a curve of longer radius than do the rear wheels. The pavement, therefore, must be wider on curves, if the clearance between passing vehicles is to remain constant.

Since the length of the vehicle is comparatively short, it is sufficient to consider that the widening per lane of traffic is equal to

the offset between the tangent and the curve in the length of the vehicle. Then from Fig. 23,

$$w = 2W = 2 \times \frac{L^2}{2R} = \frac{L^2}{R} \quad \text{(Approx.)} \tag{26}$$

where w is the widening of a two-lane road (or for each two lanes of a multiple-lane road), L is the length in feet of the vehicle measured

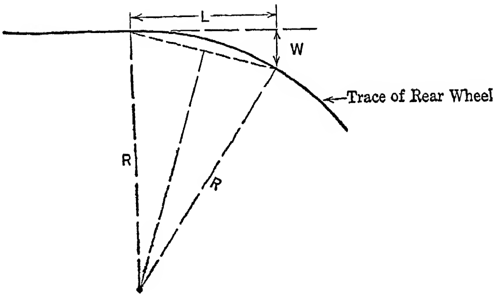


FIG. 23.

from the rear axle to the front bumper, and R is the radius of the curve in feet. It is sufficient to take R as the center-line radius. L should be about 20 ft. for ordinary conditions, and more if there are many large buses, trucks, etc.

Since it is more difficult to judge the clearance on curves than on tangents, it is desirable to have an additional width over that given by Eq. 26. This additional amount is hard to determine and is largely a matter of judgment. Mr. James T. Voshell of the U. S. Public Roads Administration suggested the following equation:[1]

but the term $(R - \sqrt{R^2 - L^2})$ is the tangent offset, hence this equation reduces to the simpler form

$$\frac{L^2}{R} + \frac{35}{\sqrt{R}} \tag{27}$$

[1] *Public Roads*, April 1927, page 35.

The second term, therefore, is an additional width to allow for the difficulty of judging the clearance.

Example. A pavement 20 ft. wide on a curve with a radius of 400 ft. is to be widened for vehicles 20 ft. long.

From Eq. 26, $w = \dfrac{20^2}{400} = 1.0$ ft., making a total required width of 21.0 ft. From Eq. 27, $w = \dfrac{20^2}{400} + \dfrac{35}{\sqrt{400}} = 2.75$ ft., making a required width of 22.75 ft., of which 1.75 ft. is "psychological" widening.

The need for widening is another argument for flatter curves, so as to reduce the amount of widening, or entirely eliminate it, and thus reduce the cost of construction.

The full amount of widening is required as soon as the entire vehicle is on the curve and hence should be required within a vehicle length of the *T.C.* and the *C.T.* of a simple curve. Obviously, it would be unsatisfactory to change the width abruptly at these points, and consequently the change of width must be made gradually in some convenient distance suitable to the character and speed of traffic.

Formerly the widening was frequently placed on the outside of the normal pavement, and the change of width was made as shown in Fig. 24*a* in a distance of 150 to 200 ft. This was objectionable, because it made reversed alinement on the outer edge, which was

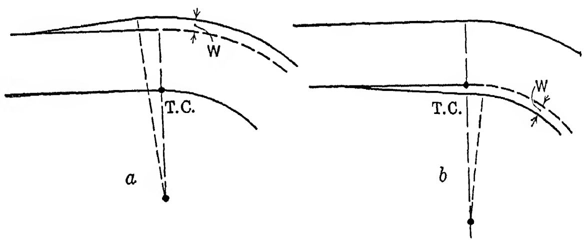


FIG. 24.

disliked by traffic and was poor in appearance. Later and better practice was to put the widening on the inner edge and make the change of width as shown in Fig. 24*b*, but this method was also objectionable on account of appearance and the difficulty of pro-

viding proper runoff for the superelevation. The best method is to place the widening on the inner edge, or symmetrically about the center line, and to make the change of width by means of spirals as explained in Chapter 5.

136. Sight Distance. On highways it is essential for safety that the drivers of vehicles be able to see approaching vehicles in time to avoid collision.

The *sight distance* is the maximum distance at which two vehicles traveling in the same traffic lane are mutually visible. Experience dictates that a sight distance of at least 700 ft. is necessary on curves, but the increase in motor-vehicle speeds indicates that not less than 1000 ft. should be provided.

The sight distance can be taken as the chord of the given curve, corresponding to a middle ordinate equal to the distance from the center line of the inner traffic lane to the object which obstructs the view. The full sight distance is not necessary at the level of the roadway, but is required at the height of the normal line of sight, which should be not more than 4.0 ft. above the roadway.

Thus in Fig. 25, M is the clearance from the center line to the

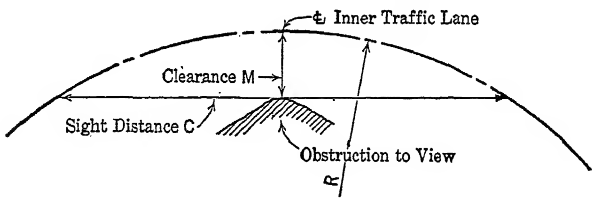


FIG. 25.

obstruction and C is the sight distance. Since the middle ordinate in the distance C is equal to the tangent offset in the distance $\frac{1}{2}C$, we have from Eq. 20

$$M = 0.0000218C^2D = \frac{0.125C^2}{R}$$

in which M is the clearance in feet, C the sight distance in feet, D is the degree of curve, and R is the radius in feet.

In designing the alinement, especially on 2-lane roads, it is highly important to provide ample sight distances at suitable intervals for passing, since some vehicles travel faster than others. If possible, such passing facilities should be provided on the tangents, but, if the topography demands a winding road, it may be necessary to

consider safe passing on curves. The flatter the curves, the more easily this can be done. The distance required to pass is not accurately known, since it depends on both the difference in speed of the two vehicles and the actual speed of the slower vehicle. The indications are, however, that from 500 to 1000 ft., in addition to the sight distances as above indicated, may be required. Thus, at as frequent intervals as possible, sight distances of one-quarter mile or more should be sought. These passing spaces should then be compared with the grade line to insure that vertical curves do not restrict them. Where the topography does not permit such provisions, immediate consideration should be given to providing at least three lanes to avoid congestion and to promote safety.

137. Safe Speeds. Since 1930, motor vehicles capable of much higher speeds than formerly have become universal, with the result that there is an upward trend in general traffic speeds. This has led to serious consideration of safe highway speeds, especially on curves.

The U. S. Bureau of Public Roads suggested that a portion of the coefficient of friction between the tires and the roadway, amounting to 0.16, might be utilized in overcoming unbalanced centrifugal force, and hence speeds in excess of the theoretical speed used in computing the superelevation could be safely permitted. The Illinois Division of Highways, after studying this proposal, reached the conclusion that an allowance of 0.16 was too great and adopted the value of 0.10. Professor Moyer* from his experiments on skidding found that, as the excess speed increased, a greater coefficient of friction was required to avoid skidding. This is another way of saying that the permissible amount of coefficient of friction to resist unbalanced centrifugal force decreases as the speed increases. His data indicate that the value varies from about 0.04 to about 0.12. Combining this idea with the usual equation for superelevation, the following equation may be derived:

$$V^2 = 15R(e + f) \tag{28}$$

where e is the superelevation as actually provided and f is the allowance in coefficient of friction to overcome unbalanced centrifugal force and may be taken as any of the values indicated above.

This equation has been erroneously considered as giving the "safe speed" on which to design new alinements. In reality it merely gives the mechanical relation between speed and a safe

*Bulletin 120, Iowa Engineering Experiment Station. 1934.

factor against lateral skidding, and fails to take into account the physio-psychological limitations of the driver and the mechanical limitations of the vehicle. This is obvious, since the equation is that of a parabola and, consequently, the speed increases indefinitely with the radius and would reach infinity on a tangent, which is an impossibility for both the car and the driver.

Several other equations have been proposed for safe speed. Some of these have been based on existing curves and, although superior to the above equation, are not reliable in speed ranges above about 50 to 60 miles per hour. Present information does not permit the derivation of a really rational equation because the safe speed on tangent is unknown. If we assume that an average car and driver can safely travel 100 miles per hour on tangent, and that a radius in excess of say 8000 feet may be considered as tangent, we may set up the elliptical equation,

$$V^2 = 2.5R - 0.000156R^2 \tag{29}$$

This equation agrees closely with the best equations for speeds, based on observed curves, up to about 70 miles per hour, and gives conservative values beyond. This equation may be used to determine a reasonably safe speed, and this value can then be used in Eq. 59 to determine the required superelevation. It will generally be found that the maximum permissible superelevation will be required, hence the curves should always be spiraled. In view of the lack of more definite information, Eq. 29, combined with maximum superelevation, is recommended for determining the minimum radius for new highways.

Problems

1. If the *P.I.* is in a stream or is otherwise inaccessible, how can I and the positions of the *T.C.* and the *C.T.* be determined?

2. If the *T.C.* is inaccessible, how can the curve be run in and checked?

3. If the line of a curve passed through a building or similar obstruction, how can the curve be run in and checked?

4. Given $I = 63°\,43'$. Find T, L, R, and E for each of the following values of D—$0°\,30'$, $2°\,00'$, $5°\,00'$, and $7°\,30'$. Solve by equation and check by Table 13.

5. Given the *P.I.* at Sta. 118 + 60.0, $I = 57°\,48'$, and $D = 5°\,00'$. Find the station numbers of the *T.C.* and the *C.T.*

6. Write transit notes for the preceding problem.

7. The $P.I.$ is at Sta. 205 + 75.0, $I = 68° 28'$. It is desired to locate a curve which will pass through a point 63.0 ft. to the right of Sta. 203 + 35.0. Find R and D.

Answer. $R = 814.2$, $D = 7° 02'$ (use $7° 00'$.)

8. The $C.T.$ of an $8° 00'$ curve is at Sta. 46 + 27.5, and the $T.C.$ of a $6° 00'$ curve is at Sta. 50 + 27.5. It is desired to eliminate the broken back by using a $3° 00'$ curve compounded with the given curves. Determine the Sta. of the $C.C._1$, the Station Equation at the $C.C._2$, and the amount the alinement is moved laterally.

Answer. Sta. $C.C.$ = 45 + 00.1; Sta. Eq. 52 + 19.9 = 52 + 17.5; $z = 18.8$ ft.

9. Given, on original location, a $6° 00'$ curve with $I = 78° 21'$ and the $T.C.$ at Sta. 1041 + 72.6. Topography requires the middle point of the curve to be moved toward the center *about* 45 ft. What is the new degree of curve and the station number of its $T.C.$?

10. Given an established curve with $I = 69° 38'$, $D = 3° 00'$, and the $T.C.$ at Sta. 982 + 41.1. It is desired to move the forward tangent *outward* a distance of 50 ft. Find the new station numbers of the $T.C.$ and the $C.T.$, and the coordinates from the old $C.T.$ to the new $C.T.$

Answer. $T.C.$ = 982 + 94.4; $C.T.$ = 1006 + 15.5; $l = 18.56$, and $p = 50.0$.

11. Data same as in Problem 10, except that the forward tangent is moved *inward.*

Answer. $T.C.$ = 981 + 87.8; $C.T.$ = 1005 + 08.9; $l = 18.56$, and $p = 50.0$.

12. Given an established curve with $I = 47° 23'$, $D = 2° 30'$, and the $T.C.$ at Sta. 1841 + 83.7. It is desired to move the forward tangent *outward* a distance of 60 ft., keeping the $C.T.$ on the same radial line *approximately.* Find the new degree of curve, the station number of the $T.C.$, and the distance that the $C.T.$ moves forward or backward due to using D to the nearest ten minutes only.

Answer. D, computed = $2° 38.7'$; D, used = $2° 40'$; $T.C.$ = 1843 + 28.1; $C.T.$ moves 7.6 ft. backward.

13. Data same as in Problem 12, except that the forward tangent is moved *inward.*

Answer. D, computed = $2° 22.2'$; D, used = $2° 20'$; $T.C.$ = 1840 + 30.3; $C.T.$ moves 16.7 ft. forward.

14. Given an established curve with $I = 50° 35'$, $D = 4° 30'$, and the $T.C.$ at Sta. 155 + 24.5. It is desired to move the initial tangent 35 ft. *inward* without changing the position of the

C.T. (except the small amount due to using approx. value of *D*). Find the new degree of curve, the station numbers of the new *T.C.* and *C.T.*, the coordinates of the new *T.C.* from the old *T.C.*, and the small amount that the *T.C.* and the *C.T.* must shift due to the approx. value of *D* used.

Answer. *D*, computed = 4° 51.8′; *D*, used = 4° 50′; *T.C.*, computed = 155 + 98.6; *T.C.*, used = 155 + 94.8; *C.T.*, computed = 166 + 38.7; *C.T.*, used = 166 + 41.3; l = 70.3; p = 35.0; *T.C.* and *C.T.* shifted 3.8 ft.

15. Data same as in Problem 14 except that the tangent is moved *outward*.

Answer. *D*, computed = 4° 11.1′; *D*, used = 4° 10′; *T.C.*, computed = 154 + 50.4; *T.C.*, used = 154 + 47.6; *C.T.*, computed = 166 + 59.1; *C.T.*, used = 166 + 61.6; l = 76.9; p = 35.0; *T.C.* and *C.T.* shifted 2.8 ft.

16. After running in a 3° 00′ curve to the right, using paper location notes, it is found that the forward tangent passes to the right of the governing point by 2° 20′ as measured at the *C.T.* (a) What distance must the *C.T.* be shifted? (b) How much does the new tangent miss the controlling point?

Answer. (a) = 77.8 ft.; (b) = 1.6 ft.

17. Given the *P.I.* at Sta. 837 + 00, I = 64° 44′, I_1 = 29° 00′, I_2 = 35° 44′, D_1 = 4° 00′, and D_2 = 5° 30′. Find the station numbers of the *T.C.*, *C.C.*, and *C.T.*

Answer. *T.C.* = 828+73.4; *C.C.* = 835 + 98.4; *C.T.* = 842+48.1.

18. Given the *P.I.* at Sta. 1846 + 50.0, I = 57° 18′, T_1 = 835.0, T_2 = 687.0, and D_1 = 3° 00′. Find I_1, I_2, D_2, and the station numbers of the *T.C.*, *C.C.*, and *C.T.*

Answer. I_1 = 16° 17.2′; I_2 = 41° 00.8′; D_2 = 4° 47.7′; *T.C.* = 1838 + 15.0; *C.C.* = 1843 + 57.9; *C.T.* = 1852 + 13.2.

19. A 2° 30′ curve compounds with a 4° 00′ curve at Sta. 8792 +27.6. The central angle of the 4° 00′ curve = 26° 45′. It is desired to move the forward tangent *inward* 30 ft., but to retain the same degree of curve. Find the station number of the new *C.C.* and the coordinates of the new *C.T.* referred to the old *C.T.*

Answer. *C.C.* = 8790 + 61.7; l = 54.5 ft.; p = 30.0 ft.

20. Data the same as in Problem 19, except that the tangent is to be moved *outward*.

Answer. *C.C.* = 8794 + 21.9; l = 66.4 ft.; p = 30.0 ft.

21. A 5° 00′ curve compounds with a 3° 00′ curve at Sta. 147 +63.3. The central angle of the 3° 00′ curve is 19° 00′. The for-

ward tangent is to be moved 25 ft. *inward*, but the same degree of curve is to be retained. Find the station number of the new *C.C.* and the coordinates of the new *C.T.* referred to the old *C.T.*

Answer. $C.C. = 149 + 03.8$; $l = 90.2$ ft.; $p = 25.0$ ft.

22. Data same as in Problem 21, except that the tangent is to be moved *outward.*

Answer. $C.C. = 146 + 61.2$; $l = 63.3$ ft.; $p = 25.0$ ft.

23. A 4° 30′ curve compounds with a 7° 30′ curve at Sta. 999 +67.0. The central angle of the 7° 30′ curve is 39° 24′. The forward tangent is to be moved *inward* 50 ft. and the *C.T.* is to be kept approximately on the same radial line. Find the new degree of curve for the second branch, the station number of the new *C.C.*, and the coordinates of the new *C.T.* (actual) referred to the old *C.T.*

Answer. D, computed = 6° 32.8′; D, used = 6° 30′; $C.C. = 996 +36.1$; $x = 5.16$ ft. forward; $p' = 47.35$ ft.

24. Data same as in Problem 23, except that the tangent is to be moved *outward.*

Answer. D, computed = 12° 51.3′; D, used = 12° 50′; $C.C. = 1003 + 31.6$; $x = 0.3$ ft. forward; $p' = 50.06$.

25. A 5° 30′ curve compounds with a 3° 40′ curve at Sta. 1888 +36.2. The central angle of the 3° 40′ curve is 27° 50′. The forward tangent is to be moved *inward* 30 ft. and the *C.T.* is to be kept approximately on the same radial line. Find the new degree of curve for the second branch, the station number of the new *C.C.*, and the coordinates of the new *C.T.* (actual) referred to the old *C.T.*

Answer. D, computed = 2° 49.0′; D, used = 2° 50′; $C.C. = 1890 + 84.4$; $x = 2.96$ ft. backward; $p' = 30.37$ ft.

26. Data the same as in Problem 25, except that the tangent is to be moved *outward.*

Answer. D, computed = 4° 03.0′; D, used = 4° 00′; $C.C. = 1886 + 01.9$; $x = 11.66$ ft. forward; $p' = 34.32$ ft.

27. What is the amount of widening required on a 7° 00′ curve for busses 30.0 ft. long, neglecting any allowance for judging clearance?

Answer. 1.10 ft.

28. What would be the widening for the curve in Problem 27 by Voshell's formula?

Answer. 2.33 ft.

29. What is the clearance required from the center line of a 20-ft. pavement on an 8° 00′ curve for a sight distance of 500 ft.?

Answer. 43.6 ft.

CHAPTER 5

SPIRALS

General Characteristics

When circular curves are used there is an abrupt change from rectilinear to curvilinear motion, and vice versa, at the *T.C.* and the *C.T.* which results in shock to both the track and the rolling stock on railroads and adds to the difficulty of steering and to the danger of skidding on highways. A similar condition exists at the *C.C.* of compound curves.

On tangents the track or roadway should be level transversely, whereas on curves the outer rail or edge of roadway should be superelevated above the inner to counteract the effect of centrifugal force. If the theoretical need for superelevation were strictly followed there would be a vertical jog in the outer rail or across the roadway at the *T.C.* and the *C.T.* which obviously is impossible from the standpoint of operation. It is necessary, therefore, to attain the superelevation in some convenient distance termed the *runoff*.

If the runoff is placed entirely on the tangent, an objectionable tilting of the vehicle occurs before the curve is reached, and, since this tilt is not balanced by centrifugal force, the vehicle drifts inward. This causes a lateral pressure on the inner rail on railroads and adds to the steering difficulties on highways. If the runoff is placed entirely on the curve, the tilting on the tangent is eliminated, but the curve is entered with no superelevation to balance the centrifugal force. The result, again, is a lateral drift, this time outward with pressure on the outer rail or with increased steering troubles for the motor vehicle. It has been customary on railroads, therefore, to place part of the runoff on the tangent and part on the curve, thereby effecting a compromise but satisfying neither condition. On highways it has been customary to place the runoff entirely on the tangents, largely for the sake of appearance, thus placing the entire burden of proper steering on the driver.

In order to overcome these objections an easement curve, or transition curve, of varying radius and of the same length as the runoff, is placed at the ends of simple curves and between the branches of compound curves in such a manner that the change of curvature is smooth and gradual and the superelevation is at all times in agreement with the degree of curve. The form of such curve commonly used is a spiral based on the cubic parabola.[1]

138. Characteristics. The spiral is best defined by stating its principal characteristics, which are:

1. The curvature of the spiral increases uniformly from its beginning to its end. At the beginning, the *T.S.*, the curvature is zero; and at the end, the *S.C.*, it is the same as that of the circular curve for which the spiral serves as a transition. If the rate of increase in curvature is 1° per 100 ft., the degree of curve of the spiral will be 1° at Sta. 1, 2° at Sta. 2, 3° at Sta. 3, and so on. On a circular curve all points have the same degree of curve, whereas on a spiral every point has a different degree of curve.

2. Offsets from the tangent at the *T.S.* to points on the spiral vary approximately as the *cubes* of the distances along the curve, instead of approximately as the *squares* of the distances as in the case of circular curves. This characteristic is illustrated in Fig. 26.

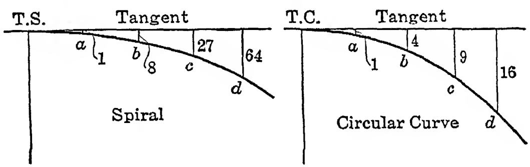


FIG. 26.

3. The deflection angles at the *T.S.* to points on the spiral vary as the *squares* of the distances along the curve instead of as the *first power* as in the case of circular curves. This characteristic is illustrated in Fig. 27.

4. The middle ordinates of successive equal chords increase uniformly as the distance along the spiral increases, instead of being

[1] Practically all of the spirals and easements curves in common use are based on this curve. They differ principally in the methods of computation and field location. The spiral here given can be substituted for any of them with no appreciable variation. For example, the so-called "10-chord spiral" is obtained by using ten chords of equal length in the formulas and methods here given.

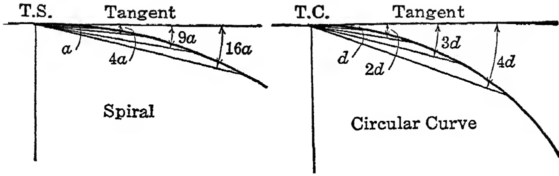


FIG. 27.

equal as in the case of circular curves. This characteristic is illustrated in Fig. 28.

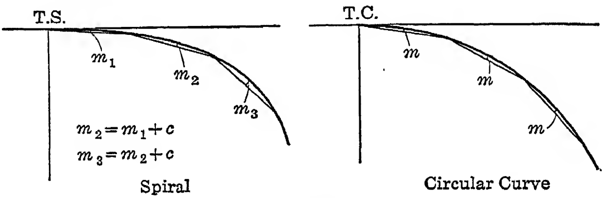


FIG. 28.

5. For a spiral, the triangle formed by the long chord as a base and the *P.I.* as a vertex is oblique, and the larger base angle is twice the smaller; whereas, for the circular curve the corresponding triangle is isosceles and the base angles are equal. This characteristic is illustrated in Fig. 29.

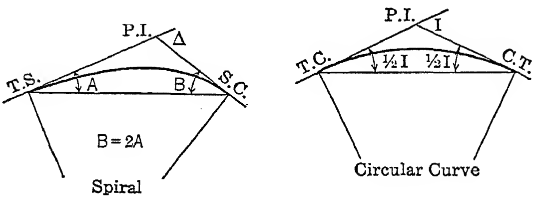


FIG. 29.

Classification of Spirals. Circular curves are classified or designated by their degrees of curve; for example, a 1° curve, a 2° curve, a 3° curve, and so on. Spirals are classified or designated by their rates of change of curvature per 100 ft. of distance. The symbol k is used to represent this rate of change. Spirals, then, are designated as $k = 1°$ spiral, $k = 2°$ spiral, $k = 3°$ spiral, etc.

139. Spiral Functions. The functions of the spiral are shown in Fig. 30, and are defined as follows:

T.S.—the point of change from tangent to spiral. The degree of curve at this point is zero.

S.C.—the point of change from spiral to circular curve. The degree of curve at this point is the same as that of the circular curve being spiraled.

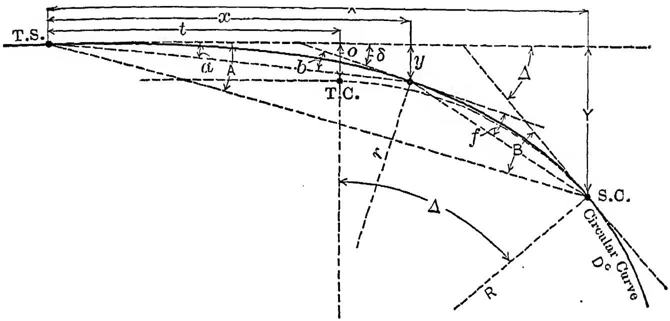


FIG. 30.

C.S.—the point of change from circular curve to spiral. It is geometrically the same as the *S.C.*

S.T.—the point of change from spiral to tangent. It is geometrically the same as the *T.S.*

a—the angle at the *T.S.* between the tangent at that point and a chord to any other point on the spiral—*the spiral deflection angle.*

A—a for the *S.C.*

b—the angle at any point on the spiral between a tangent at that point and a chord to the *T.S.*

B—b at the *S.C.*

d—degree of curve of the spiral at any point.

D—degree of curve of the circular curve to which the spiral connects.

f—the angle at any point on the spiral between a tangent at that point and a chord to any other point (b is a special case of f).

δ—the central angle of the spiral from the *T.S.* to any point.

Δ—the central angle of the spiral from the *T. S.* to the *S.C.*

k—the rate of change in the degree of curve of the spiral in degrees per station.

l—the length of the spiral in feet from the $T.S.$ to any point.

L_s—the length of the spiral in feet from the $T.S.$ to the $S.C.$

s—the length of the spiral in stations from the $T.S.$ to any point $= \dfrac{l}{100}$.

S—the length of the spiral in stations from the $T.S.$ to the $S.C.$ $= \dfrac{L_s}{100}$.

r—the radius of curvature of the spiral at any point (radius corresponding to d).

R—the radius of curvature of the circular curve to which the spiral connects.

x—the abscissa, in feet, of any point referred to the $T.S.$

X—the abscissa, in feet, of the $S.C.$ referred to the $T.S.$

y—the ordinate, or tangent offset, in feet, of any point.

Y—the ordinate, or tangent offset, in feet, of the $S.C.$

$T.C.$—the point of curve of central curve, produced back from the $S.C.$ to a tangent parallel to the initial tangent through the $T.S.$ This becomes the $C.T.$ when the stationing is in the opposite direction.

o—the ordinate, in feet, from the tangent through the $T.S.$ to the $T.C.$

t—the abscissa, in feet, of the $T.C.$ referred to the $T.S.$

$P.I.$—the point of intersection of the tangents of the spiraled curve.

$I.$—the total central angle of the spiraled curve.

T_s—the tangent distance of a symmetrically spiraled curve. ($T.S.$ to $P.I.$ and $P.I.$ to $S.T.$)

T—the tangent distance of an unspiraled curve of the same D and I.

E_s—the external distance of the spiraled curve.

E—the external distance of an unspiraled curve of the same D and I.

140. Formulas.

From definition,

$$\left.\begin{aligned} d &= ks = \frac{kl}{100} \\ D &= kS = \frac{kL}{100} \end{aligned}\right\} \qquad (30)$$

The derivations of the following formulas require the use of the calculus and are given in the Appendix.

$$\delta \text{ (in degrees)} = \tfrac{1}{2}ks^2,\ \Delta = \tfrac{1}{2}kS^2 \tag{31}$$

$$\left.\begin{aligned} a \text{ (in degrees)} &= \tfrac{1}{3}\delta = \tfrac{1}{6}ks^2 \\ a \text{ (in minutes)} &= 10ks^2, \\ A \text{ (in degrees)} &= \tfrac{1}{3}\Delta = \tfrac{1}{6}kS^2 \end{aligned}\right\} \tag{32}$$

$$\begin{aligned} b &= \tfrac{2}{3}\delta = 2a, \\ B &= \tfrac{2}{3}\Delta = 2A \end{aligned} \tag{33}$$

$$\left.\begin{aligned} y &= 0.291ks^3 - 0.00000158k^3s^7 \\ Y &= 0.291kS^3 - 0.00000158k^3S^7 \end{aligned}\right\}. \tag{34}$$

$$\left.\begin{aligned} x &= l - 0.000762k^2s^5 \\ X &= L - 0.000762k^2S^5 \end{aligned}\right\} \tag{35}$$

$$o = 0.0727kS^3 \tag{36}$$

$$t = \tfrac{1}{2}L - 0.000127k^2S^5 \tag{37}$$

$$T_s = T + o \tan \tfrac{1}{2}I + t = (R + o) \tan \tfrac{1}{2}I + t \tag{38}$$

$$E_s = E + \frac{o}{\cos \frac{1}{2}I} \qquad (R + o) \operatorname{exsec} \tfrac{1}{2}I + o \tag{39}$$

The spiral departs from any osculating circle at the same rate as from the tangent through the *T.S.* (40)

The spiral and o mutually bisect. (41)

Table 12 gives the values of L, Δ, o, t, X, and Y for values of k and D commonly used.

Application to Simple Curves

141. Field Work. Although it is possible to have spirals with different values of k at the two ends of a simple curve, there is rarely, if ever, any sound reason for so doing; hence the curves are always spiraled symmetrically.

The first step in the field work is to determine the station numbers of the *T.S.*, *S.C.*, *C.S.*, and *S.T.* The *T.S.* and the *S.T.* are located on the ground by measuring the distance T_s from the *P.I.*

From Fig 31,

$$T_s = T + o \tan \tfrac{1}{2}I + t \tag{42}$$

T is found from Table 13, o is found from Table 12 or Eq. 36, and t is found from Table 12 or Eq. 37.

From Fig. 31 it is evident that the portion of the circular curve to be retained has a central angle of $I - 2\Delta$, and hence its length is

$$L_c = \frac{I - 2\Delta}{D} = \frac{I}{D} - S \tag{43}$$

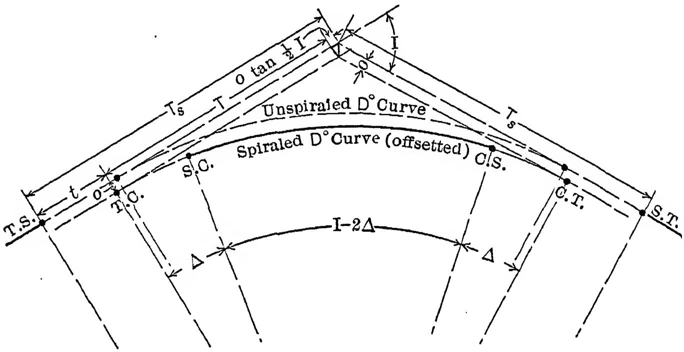


FIG. 31.

It should therefore be remembered that, in checking the deflection angles for the circular portion of the layout, the final deflection angle, for the $C.S.$, should be $\frac{1}{2}(I - 2\Delta)$.

Example. Given:—$P.I. = 21 + 21.1$, $D = 3° \ 40'$, $I = 51° \ 20'$, and $k = 1°$.

	Sta. $P.I.$ =	21 + 21.1
From Table 13,	$T = 750.9$	
From Table 12, $o = 3.58$,	$o \tan \frac{1}{2}I = \quad 1.7$	
From Table 12,	$t = 183.3$	
	$T_s =$	9 35.9
	Sta. $T.S.$ =	11 + 85.2
	S =	3 66.7
	Sta. $S.C.$ =	15 + 51.9
	L_c =	10 33.3
	Sta. $C.S.$ =	25 + 85.2
	S =	3 66.7
	Sta. $S.T.$ =	29 + 51.9

Spirals are staked out by deflection angles, by offsets, or by middle ordinates.

142. Deflection Angles. The process of staking out spirals by deflection angles is the same as that for simple curves. Spirals rarely exceed 500 ft. in length, and the total deflection angle to the *S.C.* rarely exceeds 5° 00′, therefore *the entire spiral can be run in from the T.S.*[3]

Since a spiral is more difficult to aline than a simple curve, stakes should be placed closer together; and since the spiral becomes sharper as it increases in length, stakes should be placed at shorter intervals at the end than at the beginning.

RULE:—*Place stakes* 50 *ft. apart on all spirals up to the point where the degree of curve* (d) *becomes about* 3°; *beyond this point place them* 25 *ft. apart.*

For ease in computation, stakes are placed the above distances apart *beginning at the T.S.;* hence in general *all* the stakes will fall at plusses.

The spiral deflection angles are from Eq. 32

$$a \text{ (in minutes)} = 10ks^2$$

Note that the spiral deflection angles vary with the *square* of the distance instead of the first power as in simple curves.

In Fig. 32 point 1 is located by a 50-ft. chord from the *T.S.* and the angle a_1; point 2 by a 50-ft. chord from 1 and the angle a_2; and so on, noting the change in chord length when d becomes 3°. From Eq. 32,

$$a_1 = 10'k(0.5)^2$$

$$a_2 = 10'k(1.0)^2 = 4a_1$$

$$a_4 = 10'k(1.75)^2 = \frac{49}{4}a_1$$

$$a_5 = 10'k(2.0)^2 = 16a_1$$

$$A = a_6 = 10'k(2.25)^2 = \frac{81}{4}a_1 = \frac{1}{3}\Delta$$

It is thus seen that the deflection angle for the first station only needs to be determined from Eq. 32. The other values are found by multiplying this value by the square of the ratio of the other distances to the first distance.

[3] For running in spiral from an intermediate point on spiral, see section 146.

Example. Assume that in Fig. 32, $T.S.$ is at Sta. 711 + 44.0, $k = 2$, and $S = 2.25$. Then $D = 4° 30'$.

Deflection angle of	$T.S.$,		Sta.	711 + 44.0	= 0° 00′.
"	"	" 1	"	711 + 94.0	= 0° 05′
"	"	" 2	"	712 + 44.0	= 0° 20′
"	"	" 3	"	712 + 94.0	= 0° 45′
"	"	" 4	"	713 + 19.0	= 1° 01′ (1° 01.25′ exact)
"	"	" 5	"	713 + 44.0	= 1° 20′
"	"	$S.C.(A)$	"	713 + 69.0	= 1° 41′ (1° 41.25′ exact)

$$A = \tfrac{1}{3}\Delta, \; \Delta = 5° 04'$$

$$B = \tfrac{2}{3}\Delta = 2A = 3° 22.5'$$

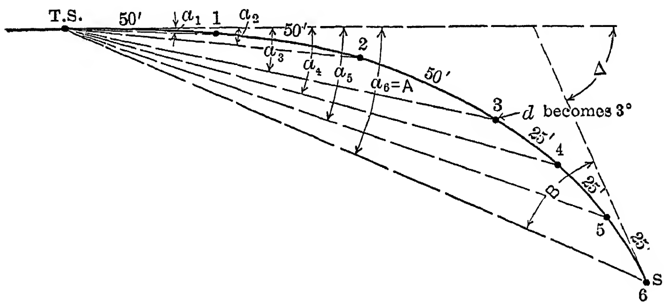


FIG. 32.

To orient the transit at the $S.C.$, back-sight on the $T.S.$ with the plates set at 3° 22.5′ (angle B); then turn the plates to zero and the telescope is on tangent and the circular curve is run in as from the $T.C.$ of a simple curve, *stakes being placed at the regular stations* (in the above problem at 714 + 00, 714 + 50, etc.). After running in about half of the circular curve, move the transit to the $S.T.$ and run in the second spiral with the same deflection angles. Then move to the $C.S.$ and *back-in* the remainder of the circular curve, thus placing the adjustment of the *errors of surveying* at the center of the curve instead of at the end of the spiral as is often but *unwisely* done. The form of notes is shown in Fig. 33.

143. Offsets. Evidently the entire spiral can be located by means of the coordinates x and y (tangent offsets), and this is a satisfactory method for short flat spirals where y is less than about 10 ft. When y becomes greater than this, the spiral can not be

located with sufficient accuracy unless the offsets are turned off with an instrument.

On location, since it is usually desirable to advance the line as rapidly as possible, the best method is to run-in the circular curve from the *T.C.* to the *C.T.* (offsetted curve, see Fig. 31), and to insert the spiral later by offsets from both the tangent and the circular curve, or by deflection angles.

Since the spiral departs from an osculating circle at any point at the same rate that it departs from the tangent at the T.S. (see Eq. 40 and sec. 265 in Appendix), it follows that the offset from the circular curve to the spiral at any distance from the *S.C.* is the same as the offset from the tangent at the *T.S.* for the same distance. Since the offset o and the spiral bisect each other (see Eq. 41 and sec. 264 in Appendix), it is evident that the maximum offset is $\frac{1}{2}o$ and it will therefore be small; also, that it will be necessary to compute offsets for half the spiral only. Half of the spiral is then located by offsets y from the tangent at the *T.S.*, and the other half by the same offsets measured normal to the circular curve.

From Eq. 34 it is seen that the offsets vary approximately as the cube of the distances, and for this method of location, and for highway spirals, they can be so considered with inappreciable error. Since the maximum ordinate is $\frac{1}{2}o$, and o is determined to offset the *T.C.*, it is seen that the offsets can be determined directly from o instead of from Eq. 34. Furthermore, it is not necessary to compute the offsets for even chord lengths, but the half-spiral can be divided into any number of equal parts and the ordinates to these points will be in the ratio of 1^3, 2^3, 3^3, 4^3, etc.

144. Middle Ordinate Method. The spiral can be staked out by the middle ordinate method. This method is especially useful in realining existing curves, where the movement of trains interferes seriously with the deflection angle method, since that method requires that the transit be set up in the center of the track.

The value of the middle ordinate of a chord of length n (in 100-ft. units), at any portion of the spiral, is approximately

$$m = 0.218\, n^2 \, (D_1 + D_2) \tag{44}$$

in which D_1 and D_2 are the degrees of curve of the spiral at the ends of the chord. For values of k less than 1.5°, this equation will give values of m which are correct to the nearest 0.01 ft.

TRANSIT NOTES FOR LINE L					
STATION	ALINEMENT	TOTAL DEFL. ANGLE	CALC. BEARING	MAG. BEARING	REMARKS
1027	Tan.		S 64°–14′ W	S 64°–10′ W	
26+23.1	S.T. ⊙	0°–00′			
+73.1		0°–02.′5			
25+23.1		0–10			
+73.1	Spiral $k=1°$ Backed in from S.T.	0–22.5			
24+23.1		0–40			
+73.1		1–02.5			
+23.1	C.S. ⊙	1–30			
+23.1	C.S. ⊙	8°–45′	$=\frac{1}{2} I_c$	(check)	
23		8–24			
22		6–54			P.I. 1020+40
21	3°-00′ C.R.	5–24			$I = 26°-30'$
1020		3–54			$D = 3°-00'$
19		2–24			$T_s = 600.'2$
18		0–54			
17+39.8	S.C. ⊙	0°–00′			
17+39.8	S.C. ⊙	1–30			$k = 1°$
+89.8		1–02.5			$L = 300.'0$
16+39.8		0–40			$\Delta = 4°-30'$
+89.8	Spiral $k=1°$	0–22.5			$A = 1°-30'$
15+39.8		0–10			$B = 3°-00'$
+89.8		0–02.5			
+39.8	T.S. ⊙	0°–00′			
1014	Tan.		S 37°–44′ W	S 37°–50′ W	

FIG. 33.

Mike McCarthy-Inst.
Heine Heinrichson-H.C.
Oley Olsen-R.C.
S.Garibaldi-R.F.

June 17, 1948
Clear-Hot

Set up at S.T.plates at $0°00'$; B.S.along Tangent;
Back in Spiral to C.S.

$I_c = I - 2\Delta = 17° - 30'$

B.S. at T.S., Plates at $3° - 00'$. Plunge and turn to
$0° - 00'$ for Tangent.

FIG. 33.

Table 3 gives the correct values of successive middle ordinates, for 100-ft. chords, for spirals whose k's are 0.25°, 0.50°, 1.00°, 1.25°, 1.50°, and 2.00°. For spirals with other values of k, the middle ordinates can be obtained by interpolation. It is to be noted that the values in the table for Sta. 0 + 50 are tangent offsets, rather than middle ordinates, as it is more convenient to locate that station by its tangent offset.

TABLE 3

Middle Ordinates for Spirals

Station on spiral	Middle Ordinates					
	k = 0.25°	k = 0.50°	k = 1°	k = 1.25°	k = 1.50°	k = 2°
	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.
$T.S.$ = 0+00	0.00	0.00	0.00	0.00	0.00	0.00
+50	0.01*	0.02*	0.04*	0.05*	0.06*	0.07*
+75	*0.01*	*0.02*	*0.04*	*0.05*	*0.06*	*0.08*
1+00	0.03	0.05	0.11	0.14	0.16	0.22
+25	*0.02*	*0.03*	*0.07*	*0.09*	*0.10*	*0.14*
+50	0.06	0.11	0.22	0.27	0.33	0.44
+75	*0.02*	*0.05*	*0.10*	*0.12*	*0.14*	*0.19*
2+00	0.08	0.16	0.33	0.41	0.49	0.65
+25	*0.03*	*0.06*	*0.12*	*0.16*	*0.18*	*0.24*
+50	0.11	0.22	0.44	0.55	0.65	0.87
+75	*0.04*	*0.07*	*0.15*	*0.19*	*0.22*	*0.29*
3+00	0.14	0.27	0.55	0.68	0.82	1.08
+25	*0.04*	*0.09*	*0.18*	*0.22*	*0.26*	*0.34*
+50	0.16	0.33	0.65	0.81	0.97	1.28
+75	*0.05*	*0.10*	*0.20*	*0.25*	*0.30*	*0.38*
4+00	0.19	0.38	0.76	0.94	1.12	1.46
+25	*0.06*	*0.11*	*0.23*			
+50	0.22	0.44	0.86			
+75	*0.06*	*0.13*	*0.25*			
5+00	0.25	0.49	0.95			

* This is a tangent offset instead of a middle ordinate.

On new work, the spiral is staked out as follows, assuming that the $T.S.$ is at Sta. 0 and using a k = 1° spiral for illustration. Sta. 0 + 50 is located by extending the tangent 50 ft. beyond the $T.S.$ and driving a stake at this point. To extend the tangent accurately, a strong, light, 100-ft. string (a silk fishline is generally used) is

stretched tightly with its rear end at a point 50 ft. back of the *T.S.* and its middle over the point at the *T.S.* A pencil mark is then made on the stake. The tangent offset, which in this problem is 0.04 ft., is then measured toward the center of the curve, and a second mark is made on the stake to give the position of Sta. 0 + 50 on the spiral. After this pencil mark has been used in the location of subsequent points on the spiral, a tack should be driven, but *not before.*

Sta. 1 is located by holding the rear end of a 100-ft. steel tape at the *T.S.* and swinging the tape in an arc until the middle point of the tape is a distance from Sta. 0 + 50 equal to the middle ordinate for that station, which in this example is 0.11 ft. It is to be noted that this value is given in Table 3 opposite Sta. 1 and is the value used in locating Sta. 1, but it is the middle ordinate measured at Sta. 0 + 50. After a stake has been driven at Sta. 1, and a 100-ft. distance from the *T. S.* marked on it, the point marking the exact location of Sta. 1 is obtained by the use of the 100-ft. string.

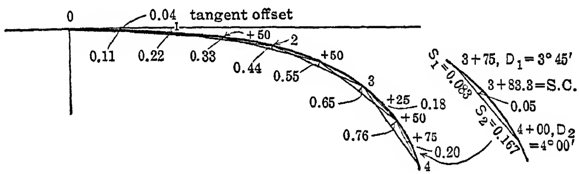


FIG. 34.

Sta. 1 + 50 is similarly located by measuring a middle ordinate of 0.22 ft. from the 100-ft. chord from Sta. 0 + 50 to Sta. 1 + 50, and by a similar procedure the remaining 50-ft. stations on the spiral are located as shown in Fig. 34. After this has been done, if it is desired to place stakes at 25-ft. intervals, the middle ordinates given in italics in Table 3 opposite the +25's and +75's are used. The procedure here, however, is somewhat different. For example, if a stake is desired at Sta. 3 + 25, a 50-ft. string is stretched between Sta. 3 + 00 and Sta. 3 + 50, and a middle ordinate of 0.18 ft. is used to locate Sta. 3 + 25.

Suppose the circular curve being spiraled is a 3° 50′ curve. With $k = 1$, the length of the spiral will be 383.3 ft. To place a stake at Sta. 3 + 83.3, the *S.C.*, a temporary stake is placed at Sta. 4, and a permanent stake at Sta. 3 + 75, as above described. The string is then stretched between Sta. 3 + 75 and Sta. 4, and Sta. 3 + 83.3 is

located by the ordinate m' at a distance of 8.3 ft. from Sta. 3 + 75. This ordinate is computed from the equation,

$$m' = 0.873\, S_1S_2 \frac{D_1 + D_2}{2} \text{ (Approx.)} \tag{44a}$$

in which S_1 and S_2 are the two segments, in stations, into which the 25-ft. chord is divided (in this case, 0.083 and 0.167), and D_1 and D_2 are the degrees of the spiral at Stas. 3 + 75 and 4 + 00, respectively, which in this problem are 3° 45′ and 4° 00′. The substitution of these values in Eq. 44a gives m' a value of 0.05 ft.

If the ground is fairly level and free from vegetation, and care is taken in stretching the string and in measuring the middle ordinates, a high degree of accuracy can be obtained by this method. The principal objection to the method, as compared with the deflection angle method, is that an error made in locating any station affects all subsequent stations and becomes larger as the distance from the point of error increases. For this reason, the deflection angle method is preferable for new work. For the realinement of existing curves, however, the middle ordinate method is the better of the two, especially if the ends of the spiral were marked by permanent monuments originally. This method is considered further under the subject of String-lining Curves in Chapter 6.

Application to Compound Curves

At the *C.C.* of a compound curve—as at the *T.C.* of a simple curve—there is a change in the rate of curvature and in the amount of superelevation, and if this is great enough to be objectionable, a spiral should be inserted between the two branches.

Evidently only that part of the spiral of curvature intermediate between the degrees of the two curves is required, and—as at the *T.C.* of a simple curve—the two curves must be offsetted at the *C.C.*

145. Problem 1. In Fig. 35, *HBC* is a D_1 curve and *EFJ* is a D_2 curve having parallel tangents at *C* and *E* (the position of the *C.C.* if the curve were unspiraled). It is desired to connect the two curves by the spiral *BF*. Consider the spiral run backward to its *T.S.* at *A*. Since the degree of curve of the spiral at *B* must be D_1 and at *F* must be D_2, then the spiral from *B* to *F* is that part of a regular spiral from where $d = D_1$ to where $d = D_2$. The value of k depends upon the maximum speed permissible on the *sharper* curve.

Since the spiral departs from every osculating circle at the same rate as from the tangent at the *T.S.*, $BC = EF$, the spiral bisects CE, and CE is equal to o for a spiral whose $D = D_2 - D_1$ for the chosen value of k.

To insert a spiral between the curves, find o for a $D_2 - D_1$ curve

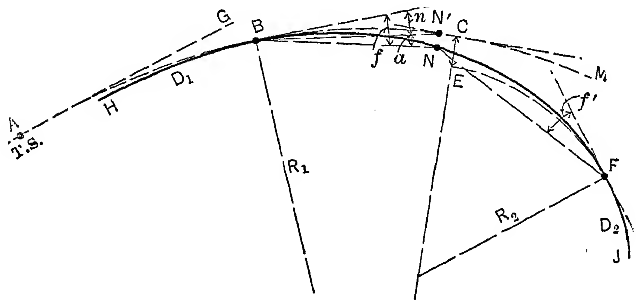


FIG. 35.

and make the offset from C to E. The length of the spiral in stations is

$$\frac{D_2 - D_1}{k} \tag{45}$$

Locate B and F by measuring $\frac{1}{2}S_1$ from C and E.

The curves may be staked out by continuing the first branch to C, offsetting to E, and running-in the second branch; and then inserting the spiral by offsets from the circular curves in exactly the same way as explained in section 143.

146. Deflection Angles may also be used, but, since the transit is at a point on the spiral, B or F, and not at the *T.S.*, the deflection angles to be used are values of f instead of a.

From a point on the tangent at B in Fig. 35, the deflection angle n to any point N' on the circular curve BCM is $\frac{1}{2}D_1 \times BN'$.

Since the spiral departs from the osculating circle at the same rate as from the tangent at the *T.S.*, the angle in degrees between the circular curve and a point N on the spiral is $a = \frac{1}{6}k(BN)^2$.

But BN and BN' are equal, whence

$$f = n + a = \tfrac{1}{2}D_1(BN) + \tfrac{1}{6}k(BN)^2 \tag{46}$$

If the transit were set at F, the deflection angle to N would be

$$f' = n - a = \tfrac{1}{2}D_2(FN) - \tfrac{1}{6}k(FN)^2$$

The *orientation angle* at F for a backsight on B is f' for the distance FB, or at B it is f for the distance BF and may be so computed. However, since the deflection angle is the same at both ends of a chord to a circle and the angle at the end of a spiral chord from the $T.S.$ is $2a$, the orientation angle is $n \pm 2a$. Therefore the simplest way to find the orientation angle is to *add or subtract, as the case may be, the last value of a to or from the last value of f ($n \pm a$) in the notes used for running the spiral.*

Example. A spiral with $k = 1°$ is to be placed between a 2° and a 5° curve with the $C.C.$ at Sta. 53 + 00 and stakes 50 ft. apart. From Eq. 45, $S = 3.000$ stations; hence the $C.S.$ at B is Sta. 51 + 50 and the $S.C.$ at F is Sta. 54 + 50. The notes are as follows:

Sta.	Pt.	Transit at B—*Read Down*			Transit at F—*Read Up*		
		n	a	$f = n + a$	n	a	$f' = n - a$
					Orient $n - 2a = 4° 30'$		
51 + 50	$C.S.$	0° 00′	0° 00′	0° 00′	7° 30′	1° 30′	6° 00′
52		0° 30′	0° 02½′	0° 32½′	6° 15′	1° 02½′	5° 12½′
+ 50		1° 00′	0° 10′	1° 10′	5° 00′	0° 40′	4° 20′
53	$C.C.$	1° 30′	0° 22½′	1° 52½′	3° 45′	0° 22½′	3° 22½′
+ 50		2° 00′	0° 40′	2° 40′	2° 30′	0° 10′	2° 20′
54		2° 30′	1° 02½′	3° 32½′	1° 15′	0° 02½′	1° 12½′
+ 50	$S.C.$	3° 00′	1° 30′	4° 30′	0° 00′	0° 00′	0° 00′
		Orient $n + 2a = 6° 00'$					

147. Problem 2. Fig. 36 shows a compound curve in which the $P.I.$, I_1, I_2, D_1, and D_2 are given. This curve is to be spiraled at the ends and between the two branches with spirals k_1, k_2, and k_3, in order. The tangent distances T_{s1} and T_{s2} are required. The solution is as follows:

From a point on the first given tangent opposite the $T.C.$ of the D_1 curve draw an auxiliary circle (shown dotted) with a radius of $R_1 + o_1$ through the angle I_1 to the $C.C.$ From this point draw a second circle with a radius of $R_2 + o_2 + o_1$ through the angle I_2 and ending in an auxiliary tangent parallel to the second given tangent at a point opposite the $C.T.$ of the D_2 curve. This auxiliary tangent

will lie outside or inside the given tangent, depending on whether $o_1 + o_2$ is greater or less than o_3, and will intersect the first given tangent at a point other than the given *P.I.* These two auxiliary circles form an ordinary compound curve concentric with the given circles as offsetted to provide room for the spirals.

The tangent distances of this auxiliary compound curve may now be computed in the usual way, as given in section 127. In triangle 1, the base is equal to the sum of the two tangent distances of the auxiliary circles, which are $(R_1 + o_1) \tan \frac{1}{2} I_1$ and $(R_2 + o_2 + o_1) \tan \frac{1}{2} I_2$, respectively, and the angles are I_1, I_2, and $180° - I$. The other two

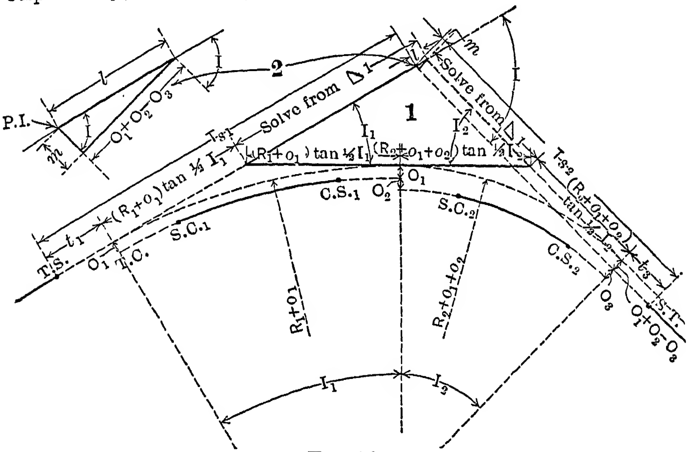


FIG. 36.

sides are computed by the sine proportion. Combining these values with the corresponding auxiliary tangent distances and the values of t_1 and t_3, there is obtained the distances from the auxiliary *P.I.*, *P′.I′*, to the *T.S.* and the *S.T.* It is now necessary to correct these distances to the given *P.I.* To do this, form triangle 2 by dropping a perpendicular from the auxiliary *P.I.* to the second given tangent. In this triangle the base is $o_1 + o_2 - o_3$ and the given angle is *I*. Compute the sides *l* and *m*.

If the auxiliary tangent lies outside the given tangent, $o_1 + o_2 - o_3$ is positive, as shown in Fig. 36, and the side *l* is subtracted from the computed distance to the *T.S.* to obtain the desired T_{s1}. At the same time, the side *m* is added to the computed distance to the *S.T.* to obtain T_{s2}.

If the auxiliary tangent lies inside the given tangent, and this occurs when o_3 is greater than $o_1 + o_2$, then $o_1 + o_2 - o_3$ is negative, and l and m are applied with signs opposite to those above.

If I is greater than 90°, triangle 2 changes position with respect to the *P.I.*, and the signs of l and m are opposite to those indicated above in each case. To avoid mistakes, however, it is always desirable to draw a sketch of this triangle in proper position with respect to the *P.I.* and determine by inspection the proper signs for l and m.

Multiple-centered compound curves can be computed by an extension of this method.

Railroad Spirals

148. Choosing S and k. Since the function of the spiral is to ease the entrance to a circular curve, D will always be known. Then in Eq. 30, it is necessary only to choose a value of k or S, and the spiral is fixed. Obviously, any value of k could be chosen and the spiral would fit, but such a spiral may be so short as to require an excessive rate of attaining the superelevation, or it may be needlessly long. Similarly any value of S might be chosen and the spiral would fit, but in this case an odd value of k is likely to result, and this complicates the calculation and the field work. The logical method, therefore, is first to determine a satisfactory length of spiral to give a suitable rate of superelevation, and from this to compute an approximate value of k from which a simple value, convenient for computations and field work, may be chosen.

The length of the runoff, and hence the length of the spiral, is a direct function of the total superelevation and of a rate of attaining this superelevation that will be comfortable to passengers. The total superelevation for standard-gage track depends upon the speed and the degree of curve. It is given by the equation,

$$e^* = 0.00069\, DV^2 \qquad (47)$$

in which e is the superelevation in inches, D is the degree of curve, and V is the speed in miles per hour. The value of e should not exceed about 8 in., on account of slowly moving trains, but the track should be superelevated for the fastest trains up to this limit. The American Railway Engineering Association states that curves requiring a superelevation of less than 2 in. need not be spiraled, but many railroads spiral for superelevations of 1 in. With the coming of the

* For derivation, see Appendix, section 266.

high-speed "streamliners" there has been a distinct tendency to superelevate for lower values of e, down to $\frac{1}{2}$ in.

The maximum rate at which superelevation may be attained without discomfort to passengers may be taken as $1\frac{1}{6}$ in. per second. Therefore the minimum desirable length of runoff is

$$L = \tfrac{6}{7}\, ev$$

where v is the velocity in feet per second. Reducing v to V, the speed in miles per hour,

$$L = 1.26\, eV$$

Substituting the value of e from Eq. 47, and changing D to R,

$$L = 0.00087\, DV^3 = \frac{5V^3}{R} \text{ (approximate)} \tag{48}$$

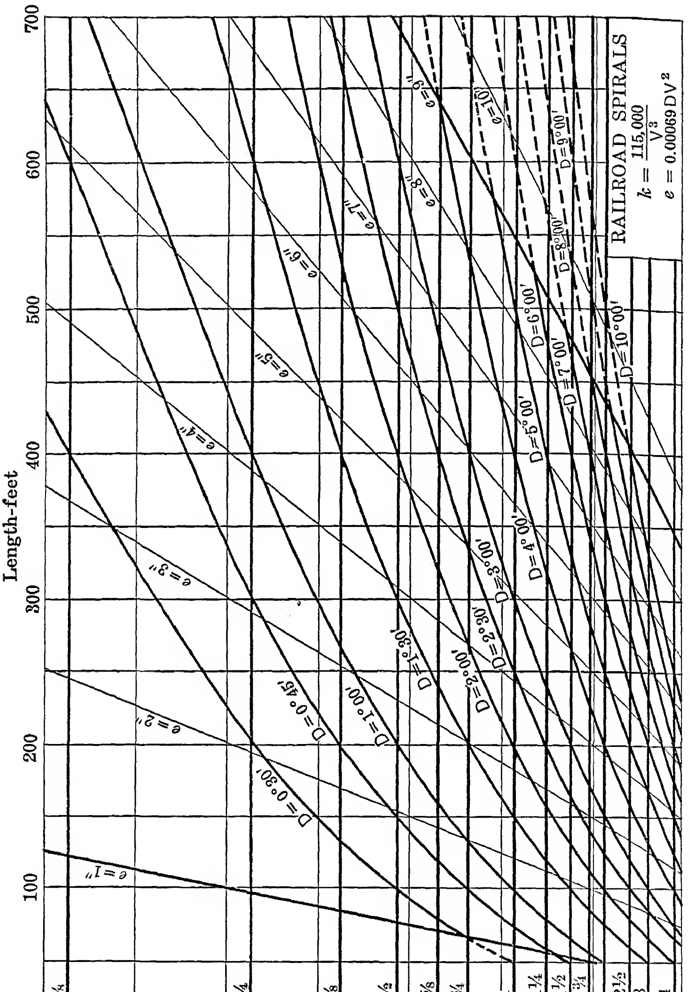
Substituting the value of D from Eq. 30 and solving for k,

$$k = \frac{115{,}000}{V^3} \text{ (approximate)} \tag{49}$$

It is evident, therefore, that a suitable value of k can be determined directly when the speed of operation is known. Consequently the proper procedure in any given case is to determine first the normal maximum speed to be expected. Then from Eq. 49 compute a trial value of k from which a simple value, convenient for computation and field use, is selected.

The diagram, Fig. 37, is plotted from values computed by the preceding equations. It gives k, L, and e for curves from 0° 30′ to 10° 00′ and speeds from 30 to 100 miles per hour. From this diagram, with any two of the five values, D, V, k, e, or L given, the other three may be determined within the accuracy of scaled values. It is especially useful in determining k and e when D and V are known. The relation between k and L should always be determined from Eq. 30 for the sake of accuracy.

Examples. (*a*) A 4° curve is to be operated at 50 miles per hour. Entering the diagram with $V = 50$ and following the horizontal line to its intersection with the curved line marked $D = 4°\ 00'$, we can read directly that the superelevation should be about 7 in., that the theoretical length should be a little more than 425 ft., and that k should be a little less than 1°. The nearest convenient value of k is 1°, and, when this value is substituted in Eq. 30, L is found to be



RAILROAD SPIRALS
k = 115,000 / V³
e = 0.00069 DV²
Length-feet
Speed-miles per hour

FIG. 37.

400 ft., which is verified by the diagram. Thus the curve should be spiraled with a $k = 1°$ spiral 400 ft. long, and the superelevation should be 7 in. to agree with the speed of 50 miles per hour.

(*b*) A given $2° 30'$ curve is superelevated 6 in. Following the curved line marked $2° 30'$ to its intersection with the inclined line, marked $e = 6$ in., we find that V is about 59 miles per hour, and L is 450 ft. From Eq. 30 the value of k corresponding to $L = 450$ is 0.5555, which is an inconvenient value to use. The diagram shows that the nearest convenient value is $k = \frac{1}{2}$, which would be used, giving a spiral length of 500 ft.

(*c*) It is proposed to use a superelevation of 4 in. with a runoff of 300 ft. From the intersection of the $e = 4$-in. line with the $L = 300$-ft. line, we find that the speed is about 60 miles per hour, the degree of curve is $1° 30'$, and the value of k is $\frac{1}{2}°$.

149. Variation in Speeds. Since the centrifugal force due to curvature varies with the square of the speed, while the amount of superelevation in the track remains fixed, it follows that a curve can be theoretically superelevated for only one speed, although the trains will actually operate at different speeds.

If the speed is below that for which the curve is theoretically superelevated, there will be an unbalanced, inward component which may be increased somewhat by a component of the drawbar pull. This force must be resisted by flange pressure against the inner rail. This force will tend to overturn the rail; hence rail braces are frequently used on the inside rail of heavily superelevated tracks. At the same time this pressure tends to move the track inward and often results in putting the track out of alinement. Also, in case of slowly moving passenger trains, the unbalanced tilt may be objectionable to passengers. These are the reasons for limiting the maximum superelevation to about 8 in.

If the speed is greater than that for which the track is superelevated, the centrifugal force exceeds the component due to slope, and a pressure on the outer rail results. Thus, rail braces are often found also on the outside rails to resist the tendency for the rail to overturn. This pressure also tends to throw the track out of line. When this is combined with the tendency for slow trains to move the track inward, it often happens that the track gets out of alinement very quickly.

The theoretical speed for which the track is superelevated is known as the "equilibrium speed." Trains may, however, operate at a maximum speed in excess of the equilibrium speed. According to

the American Railway Engineering Association, the maximum speed may exceed the equilibrium speed by an amount equal to an increase of 3 in. in the superelevation. For example, a track superelevated 6 in. may be safely operated at a speed requiring a theoretical superelevation of 9 in.

Application to Existing Curves

To insert a spiral in an existing track, it is necessary to shift the ends of the curve inward to provide room for the spiral. If the same degree of circular curve were retained, this would shift the *entire curve* inward. Since the amount of such shifting may be con-

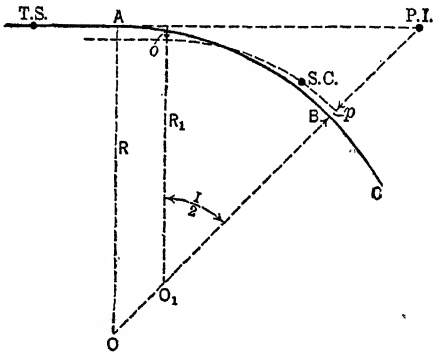


FIG. 38.

siderable (o sec $\frac{1}{2}I$ at its center) the new alinement may not be on the old roadbed and considerable cost of earthwork would be entailed in making the change. To obviate this, the degree of curve may be changed in such a way that the new alinement will permit the insertion of the spiral and at the same time require little or no additional earthwork.

150. SIMPLE CURVES. Existing simple curves may be spiraled in two ways.

(1) By shifting the center of the curve *outward*[4] a small amount, and by sharpening the curve sufficiently to give the desired offset. This method is particularly applicable for curves whose lengths are less than about four times the length of the spiral to be used.

[4] The shifting must be *outward* to make the change in alinement a minimum.

(2) By sharpening the curve slightly at the ends to give the necessary offset, leaving the middle portion of the curve unchanged. This method is preferable for long curves.

151. First Method.

In Fig. 38, ABC is the existing unspiraled curve with B as its middle point. It is desired to shift the track *outward* at B the *assumed* distance p, and to sharpen the curve to give the necessary offset o.

From the figure, p is the difference between the external distances of the existing curve and the spiraled curve. Therefore from Eqs. 11 and 39, we have

$$p = R \text{ exsec } \tfrac{1}{2}I - [(R_1 + o) \text{ exsec } \tfrac{1}{2}I + o]$$

$$= (R - R_1 - o) \text{ exsec } \tfrac{1}{2}I - o \qquad (50)$$

In Eq. 50, both R_1 and o are unknown, but $o = 0.0727 \dfrac{(5730)^3}{R_1{}^3k^2}$

This value of o could be substituted in Eq. 50 and an expression for R_1 found, but obviously the expression would be a difficult one to use. It is faster and simpler to solve the equation by successive approximations as follows: Choose a value of R_1 less than R (preferably to agree with an even 10 minutes of degree of curve) and solve Eq. 50 for p. If this value agrees sufficiently closely with the desired shifting of the track at B, the solution is complete. If not, re-estimate R_1 (using the previous value as a guide), and repeat until a satisfactory value of p is obtained.

The distance from the $T.C.$ of the original curve to the $T.S.$ is the difference between T_s for the new curve and T for the old curve. The $T.S.$ and the $S.T.$ are then located from the $T.C.$ and the $C.T.$, respectively, of the old curve, and the new curve is staked out as in new work.

Example. $I = 40°\,00'$, $D = 4°\,00'$, $k = 1°$, and $p =$ about 1.5 ft.

Choose a trial value of $R_1 = 1322.3$ ft. for $D_1 = 4°\,20'$. Then from Eq. 50, $p = 0.8$ ft. which is less than that desired. Choosing $R_1 = 1312.2$ ft. for $D_1 = 4°\,22'$, p becomes 1.3 ft., showing that a change of 2′ in D_1 increases p about 0.5 ft. Then to make $p = 1.5$ ft., D_1 by interpolation would have to be about 4° 23′. Since an odd value of D_1 is undesirable, and since it is not necessary to make p exactly 1.5 ft., it is sufficient to take $D_1 = 4°\,22'$.

In the foregoing discussion it was assumed that the degree of the existing curve was known and that the $T.C.$ and $C.T.$ were monumented. It will generally happen, however, that the degree

of curve is unknown, that the track is in poor alinement, and that the *T.C.* and *C.T.* are not monumented. In this case, run out the tangents to an intersection and measure the intersection angle. Then measure the external distance of the existing curve and from it determine the degree of the curve that will connect the tangents and pass through the middle point of the existing curve. Use the value of R corresponding to this value of D in solving Eq. 50.

152. Second Method. Case 1. In Fig. 39, *ABC* is the existing unspiraled curve, whose degree of curve, D, is known, whose alinement is good, and whose *T.C.* and *C.T.* are monumented. At some point on this curve, such as B, it is desired to compound with an assumed curve of slightly shorter radius, R_1, which, when run to a tangent parallel to the initial tangent, will be a distance from it equal to o for a R_1 curve, thus providing room for the spiral.

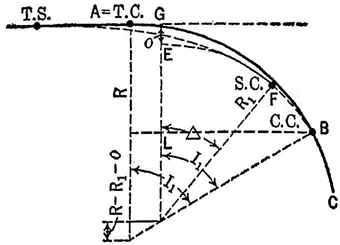


FIG. 39.

In the figure, *EFB* is the R_1 curve, and *EG* is o corresponding to it. It is required to find the location of the *T.S.*, of the *S.C.*, and of the *C.C.*, *B*.

From the figure,

$$o = EG = GL - EL = (R - R_1) \text{ vers } I_1$$

and

$$\text{vers } I_1 = \frac{o}{R - R_1} \tag{51}$$

The *T.S.* is located by measuring back from the old *T.C.*

$$T.C. \text{ to } T.S. = t - (R - R_1 - o) \tan I_1 \tag{52}$$

The *S.C.* is located by measuring the spiral length from the *T.S.* The *C.C.*, *B*, is located by measuring the distance *FB* from the *S.C.*

$$FB \text{ (in feet)} = \frac{I_1 - \Delta}{D_1} 100 \tag{53}$$

The location of the point B may be checked by measuring the distance *AB* along the old curve.

$$AB \text{ (in feet)} = \frac{I_1}{D} 100$$

The distance from the $T.S.$ to the $C.C.$ along the new alinement is shorter than along the old alinement, and the track will have to be shortened. This shortening is equal to the difference in the station numbers of the $C.C.$ as computed along the respective alinements.

The limits of R_1 are such as will make the point B come at the middle of the original curve, or will make B and F coincide. In general, R_1 should be chosen between $0.8R$ and $0.9R$.

Example. $D = 3° \ 30'$, D_1 (assumed) $= 4° \ 00'$, $k = 1°$, and $o = 4.65$. From Eq. 51, $I_1 = 12° \ 14.3'$. From Eq. 52 the distance

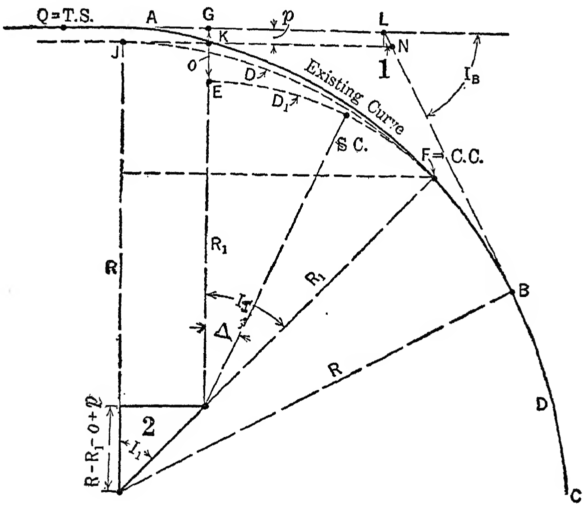


FIG. 40.

from the old $T.C.$ to the $T.S.$ is 156.5 ft. The spiral is 400 ft. long. From Eq. 53, the distance from the $S.C.$ to the $C.C.$ is 106.0 ft. The new alinement will be about 0.3 ft. shorter than the old alinement.

153. Second Method. Case 2. In Fig. 40, ABC is the existing unspiraled curve, whose degree of curve is unknown, whose alinement may be poor, and whose $T.C.$ and $C.T.$ are not monumented. First set-up in the center of the track at some point C near the middle of the curve and by trial deflection angles find the degree of the curve, D, that will most nearly conform to the existing track. Then run-in this D curve to a point B which is about 500 ft. from the end of the curve. If this curve were continued, its $T.C.$ would fall at J

on a tangent parallel to the initial tangent QL and at a distance p from it. At B run-out the tangent to the D curve to an intersection with the initial tangent at L and measure the intersection angle I_B and the distance BL. BN is the tangent distance of the curve corresponding to I_B. Then, in triangle 1,

$$p = NL \sin I_B$$

The D curve is to be compounded with a chosen D_1 curve at some point F to give the required offset o.

$$EK = o - p = (R - R_1) \operatorname{vers} I_1, \text{ whence}$$

$$\operatorname{vers} I_1 = \frac{o - p}{R - R_1} \tag{54}$$

The $T.S.$ is located at Q by measurement from L.

$$QL = QG + GL = t + JN - JK - NL \cos I_B$$

The $S.C.$ and the $C.C.$ are located as in Case 1, and the spirals are staked out as in new work.

J may fall outside of QL, in which case p and o are numerically added; again, p may be greater than o, and in this case R_1 must be greater than R.

154. COMPOUND CURVES.

An existing compound curve can be spiraled in the following manner.

In Fig. 41, ABC is the existing compound curve with its $C. C.$ at B. To make room for the spiral it is necessary to compound the sharper branch, D_2, at some point C with a still sharper curve of chosen degree, D_3, and to continue the first branch, D_1, to the point E where its tangent is parallel to the tangent of the D_3 curve at G. EG is the offset o for a spiral whose $D = D_3 - D_1$. The problem is to find $BE = n_1$ and $CG = n_2$.

For small central angles the ordinates between two circular curves are equal to the difference in their tangent offsets. Therefore EF can be considered as the difference in the tangent offsets of the D_1 and D_2 curves in the distance n_1; similarly FG is the difference in the tangent offsets of the D_2 and D_3 curves for the distance n_2. It must be remembered that the figure is very much exaggerated.

Then from Eq. 20,

$$o = EF + FG = 0.873(D_2 - D_1)n_1^2 + 0.873(D_3 - D_2)n_2^2.$$

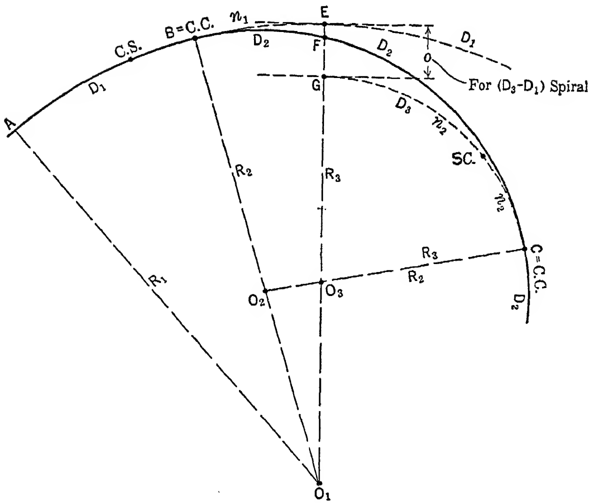


FIG. 41.

Since the central angle of the arc BC equals the sum of the central angles of the arcs BE and CG,

$$D_2(n_1 + n_2) = D_1n_1 + D_3n_2$$

or

$$(D_2 - D_1)n_1 = (D_3 - D_2)n_2.$$

Solving these simultaneous equations for n_1 and n_2 we have

$$= 1.07\sqrt{\frac{(D_3 - D_2)o}{(D_2 - D_1)\,(D_3 - D_1)}} \qquad (55)$$

$$n_2 = 1.07\sqrt{\frac{(D_2 - D_1)o}{(D_3 - D_2)\,(D_3 - D_1)}} \qquad (56)$$

C is located by measuring the distance $n_1 + n_2$ along the D_2 curve from B. The $C.S.$ is located by running the D_1 curve from B the distance $n_1 - \frac{1}{2}S_1$, and the $S.C.$ is located by running the D_3 curve from C the distance $n_2 - \frac{1}{2}S_1$. The spiral is staked out as usual for compound curves.

Highway Spirals

Spirals seem to have been first used on highways merely to run out the changes in width when pavements were widened on curves. But as motor vehicle speeds increased until they equaled or exceeded

train speeds, it began to be realized that highway spirals were distinctly desirable. Indeed a careful analysis reveals the fact that spirals are even more necessary on highways than on railroads because the motor vehicle is steered by the driver instead of being guided by the rails.

No motor vehicle can change instantly from rectilinear to curvilinear movement. A finite interval of time is required to turn the steering wheel, during which the car moves forward; and the natural action of the driver is to turn the wheel at a rate that is convenient and that makes smooth riding. If it is assumed that the wheel is turned at a uniform rate, it follows that the angular change of direction is increasing at a constant rate, which may be designated k. If the car moves forward at a constant speed, the total change of direction will vary with this change in angle and the distance traveled. Consequently,

$$\delta = \int ks\, ds = \tfrac{1}{2}ks^2$$

in which δ is the total change of direction in the distance s, and k is the unit rate of change in the steering angle.

This equation is identical with Eq. 31, which gives the total change of direction of the spiral, and shows that the natural path of the vehicle under the natural process of steering is a spiral.

Another important factor in determining the path of the car and the need for spiraling is the effect of superelevation and the manner in which it is introduced. To analyze this factor, consider a car approaching an unspiraled curve at the speed for which the curve is superelevated and with all the runoff on the tangent, as is common practice.

On normal tangent the pavement is level transversely; hence the only lateral forces acting on the car are those incident to normal driving and these are met by the ordinary processes of steering. But, as the car traverses the runoff, it is subjected to an increasing lateral force due to the increasing tilt of the pavement. Since the alinement is straight, there is no balancing centrifugal force; hence this lateral force tends to cause a lateral inward "drift" of the car. This force can by no means be neglected, since it amounts to 20 lb. per ton for each percent of cross slope. Thus for a superelevation of 0.10 ft. per ft. of width, it amounts to 200 lb. per ton, or about four times the tractive effort required to drive the car at 50 m.p.h. on an average level road.

If the driver wishes to stay in his proper lane, he must overcome this lateral force by steering against it. This steering movement is outward, or opposite in direction to that required by the approaching curve, and hence is not a natural steering movement. Consequently, few drivers do it, but permit the car to take the natural drift inward. This lateral force due to the tilt is resisted only by a component of the "rolling resistance" of the road. Since the total drift occurs in the total length of the runoff, it is made at a very small angle with line of travel, and consequently the total value of the component is small and may be neglected.

Since the lateral force is a direct component of gravity, and the tilt is increasing at a uniform rate, we may write $df = n\,dx$, where n is a constant depending on the total superelevation and length of runoff. From the principles of mechanics, we know that $df = m\,da$ and, also, that the movement caused by an acceleration is $dy = \frac{1}{2}t^2da$, where t is the time. Since the car is moving at a constant speed, it follows that $t = \frac{x}{v}$. Combining these equations we find that

$$y = \int \frac{nx^2}{2\,mv^2}\,dx = \frac{nx^3}{6\,mv^2} \tag{57}$$

where y is the lateral drift in feet in the distance x in feet, n is the unit increase in the tilt (equal to the total superelevation in ft. per ft. of width divided by the length of runoff in feet), m is the unit of mass (equal to $\frac{1}{32}$), and v is the speed of the car in feet per second.

Obviously this is the equation of a *cubic parabola*, and it is well known that the common transition spiral is based on the cubic parabola. It is thus shown that the natural effect of placing the runoff on the tangent is to cause the car to tend to move in a curved path that is essentially a spiral.

It has already been shown that the natural steering process also tends to cause the car to move on a spiral. That this is true is abundantly shown by the frequency with which cars run off the pavement on the inside at the entrance to curves to the right and encroach on the adverse traffic lane at the entrance to curves to the left, with many consequent accidents and fatalities.

If a spiral of the same length as the runoff had been used, the increasing lateral force due to the increasing tilt would have been balanced by an equal centrifugal force due to the increasing curvature. The effect of the unbalanced tilt would thus have been eliminated, and the driver would have been relieved of any necessity

of trying to compensate for it, while the spiral alinement would have conformed to his natural steering practice. Thus, the introduction of the spiral would have simplified the steering process and at the same time promoted comfort and safety by eliminating the tendency to lateral drift. *The use of the spiral simply means that the road is built on the path which the car tends to follow under the forces acting on it and under the natural process of driving.*

As an example, assume a 2° curve superelevated 0.10 ft. per ft. of width, with a runoff distance of 200 ft. and a car speed of 60 m.p.h. Then $n = 0.0005$, $x = 200$, $m = \frac{1}{32}$, and $v = 88$. Substituting these values in Eq. 57, $y = 2.75$ ft. This is sufficient, if uncompensated in the steering, to cause the car to encroach dangerously on the adverse lane or to run off the pavement. On the other hand, if a spiral with $k = 1°$, giving the same length of runoff, had been used, Y at its end would have been 2.33 ft. (Table 12). This is only 0.42 ft. different from the natural path of the car, instead of 2.75 ft. as in the case of the unspiraled curve, and this small variation is permissible in the ordinary width of traffic lane. In traversing the spiral the driver would have 200 ft. (equivalent to about $2\frac{1}{4}$ seconds at the given speed) in which to turn his wheel at a gradually increasing rate; hence the steering would be a simple, natural, and convenient process.

If a spiraled curve is approached at a speed less than that for which it is superelevated, part of the force due to tilt will not be balanced by centrifugal force, which will result in some tendency of the car to drift inward. To overcome this drift the driver must steer against it by turning the steering wheel a less amount than required by the curvature. The net movement, however, will be in the direction of curvature and, with the entire length of spiral in which to accomplish it at a gradual rate, the entire steering process will be done in a simple and natural manner. On the other hand, if the spiraled curve is approached at a speed greater than that for which it is superelevated, part of the centrifugal force will not be balanced by tilt; hence there will be some tendency for the car to drift outward. To overcome this the driver must turn his wheel more than required by the curvature. But again the steering movement is all in the direction of curvature with the entire length of spiral available in which to make it gradually; hence the steering process is again simple, natural, and convenient. Thus, whether the speed be above, below, or at the theoretical speed, the steering is simplified, and consequently the curve is made safer and more convenient.

155. Choosing D or R. Horse-drawn traffic permitted the use of very sharp curves, whence arose the practice of designating highway curves by the radius instead of the degree of curve. Although modern traffic requires flatter curves, the custom of designating curves by the radius still persists, even though the degree of curve is more desirable for exactly the same reasons as on railroads. For all radii greater than about 300 ft., a simple value of the degree of curve should be chosen, rather than an even value of the radius.

The use of the radius instead of the degree of curve does not, however, greatly complicate the use of the spiral. From Eq. 30, $d = ks$, and from Eq. 15, $d = \frac{5730}{r}\cdot$ Substituting the latter in the former, we have

$$r = \frac{5730}{ks} \quad \text{and} \quad R = \frac{5730}{kS} \tag{30a}$$

Equation 30 is used when D is chosen, and Eq. 30a is used when R is chosen.

156. Choosing S or k. It has been the common practice in highway work to choose the length of runoff, and thus the length of spiral, and then, with D or R known, to determine the other factors. This generally gives odd values of k, which are distinctly inconvenient in both computations and field work. It is better to determine a simple value of k and permit S to be odd, if necessary, since this method simplifies both computations and field work.

Owing to the manner of steering, motor vehicles do not change abruptly from rectilinear to curvilinear motion, but each provides its own spiral to a limited extent, as previously pointed out. Furthermore, recent investigations show that motorists will readily accept a higher rate of tilt and more unbalanced centrifugal force than will passengers on a train. These factors tend to reduce the required length of spiral, or, what is the same thing, permit a higher value of k.

Professor Moyer * from some extended investigations of skidding of automobiles arrived at the conclusion that the minimum length of highway spiral is given by the equation

$$L_s(\text{in feet}) = \frac{1.58\,V^3}{R}$$

* Bulletin 120, Iowa Engineering Experiment Station, Ames, Iowa, 1934.

which is equivalent to a value of k of

$$k = \frac{363000}{V^3}$$

Professor Moyer distinctly states that this is the sharpest spiral that should be employed. Consequently, a spiral of greater length, or lower value of k, should be used wherever possible.

From a study of Moyer's work, railroad practice, and existing highway spirals, the authors recommend that the following equation be used to determine the approximate *desirable* value of k.

$$k = \frac{230000}{V^3} \tag{58}$$

This equation gives values of k twice as great as for railroads, or spirals one-half as long. It provides spirals about 58% longer than the minimum suggested by Moyer. Fig. 42 is a highway spiral diagram based on Eq. 58 and is used in the same manner as Fig. 37.

Example. Assume that a curve with a radius of 800 ft. is to be operated at 40 m.p.h. From Moyer's equation the minimum length of spiral would be

$$L_s = \frac{1.58 \times 40^3}{800} = 126.4 \text{ ft.}$$

From Eq. 58, the approximate desirable value of k would be

$$k = \frac{230000}{40^3} = 3.6$$

from which the more convenient value of $k = 3.5$ would be chosen. The length of the spiral from Eq. 30a would be

$$L_s = 100S \quad \frac{573000}{3.5 \times 800} = 204.6 \text{ ft.}$$

Assume, however, that the radius of 800 ft. with a spiral length of 200 ft. was chosen with stakes to be located at 50-ft. intervals; then from Eq. 30a,

$$k = \frac{5730}{800 \times 2} = 3.58125$$

From Eq. 31,

$$\Delta = \tfrac{1}{2} \times 3.58125 \times 2^2 = 7.1625^\circ = 7^\circ\, 09.75'$$

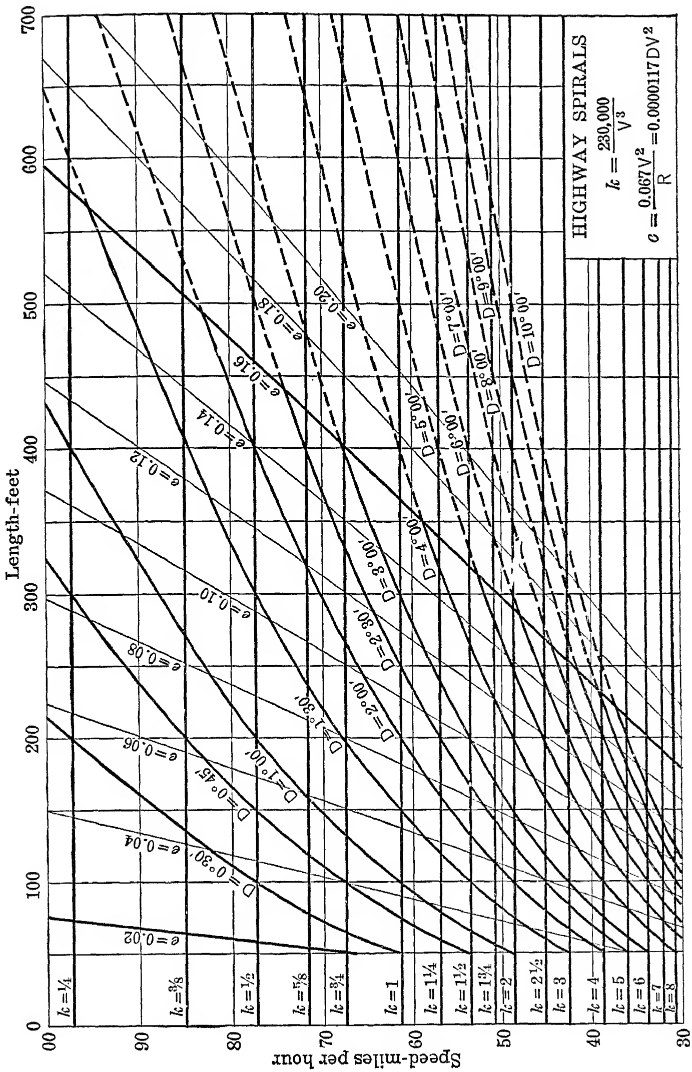

HIGHWAY SPIRALS
$k = \frac{230,000}{V^3}$
$e = \frac{0.067V^2}{R} = 0.0000117DV^2$
Length-feet
Speed-miles per hour

FIG. 42.

From Eq. 32,

$$
\begin{aligned}
a_1 &= 10 \times 3.58125 \times 0.5^2 &&= 0^\circ\, 08.953' \\
a_2 &= 2^2 \times 8.953 &&= 0^\circ\, 35.81' \\
a_3 &= 3^2 \times 8.953 &&= 1^\circ\, 20.58' \\
a_4 &= 4^2 \times 8.953 &&= 2^\circ\, 23.26' = A = \tfrac{1}{3}\Delta
\end{aligned}
$$

From Eq. 15, $D = \dfrac{5730}{800}$ - 09.75′ on which the deflection angles for the circular curve must be based.

If k had been chosen as 3.5, then

$$S \quad \frac{5730}{3.5 \times 800} = 2.0464 \quad \text{and} \quad \Delta = 7\;\,19.6'$$

$$
\begin{aligned}
a_1 &= 10 \times 3.5 \times 0.5^2 &&= 0^\circ\, 08.75' \\
a_2 &= 2^2 \times 8.75 &&= 0^\circ\, 35.00' \\
a_3 &= 3^2 \times 8.75 &&= 1^\circ\, 18.75' \\
a_4 &= 4^2 \times 8.75 &&= 2^\circ\, 20.00' \\
a_5 &= A = \tfrac{1}{3}\Delta &&= 2^\circ\, 26.53'
\end{aligned}
$$

The circular deflection angles would be the same as before, but the spiral deflection angles have been simplified.

Assume that a $7^\circ\, 00'$ curve, which has approximately the same radius, had been used with $k = 3.5^\circ$. Then $S = 2.0$ and $\Delta = \frac{1}{2} \times 3.5 \times 2^2 = 7^\circ\, 00'$. a_1, a_2, and a_3 would be the same as in the last example while $a_4 = A = \frac{1}{3}\Delta = 2^\circ\, 20'$, and the circular deflections would be for a 7° curve. This shows that even values of D and k are more desirable than even values of the radius and length of spiral. Table 11 gives the necessary data for even radius curves.

157. Superelevation. *The theoretical superelevation for highways is given by the equation*

$$\frac{0.067V^2}{R} = 0.0000117DV^2 \tag{59}$$

where e is the superelevation in feet per foot of width, V is the speed in miles per hour, and R and D are the radius and the degree of curve, respectively. Table 4 gives the theoretical superelevation for several different radii and speeds.

TABLE 4.—THEORETICAL SUPERELEVATION. Transverse slope per foot of width $e = 0.067\ V^2/R$.

Speed, miles per hour	Radius, feet (above); degree of curve (below)																		
	50	60	75	100	150	200	300 19°06′	400 14°20′	500 11°28′	600 9°33′	700 8°12′	800 7°10′	900 6°22′	1,000 5°44′	1,500 3°49′	2,000 2°52′	3,000 1°55′	5,000 1°09′	10,000 0°34.4′
5	7/16 **0.034**	5/16 **0.028**	1/4 **0.022**	3/16 **0.017**	1/8 **0.011**	1/8 **0.008**	1/16 **0.006**												
10	1 5/8 **0.135**	1 5/16 **0.112**	1 1/8 **0.090**	13/16 **0.067**	9/16 **0.045**	7/16 **0.034**	1/4 **0.022**	3/16 **0.017**	3/16 **0.013**	1/8 **0.011**	1/8 **0.010**	1/8 **0.008**	1/16 **0.007**	1/16 **0.006**					
15	3 5/8 **0.303**	3 **0.253**	2 7/16 **0.202**	1 13/16 **0.151**	1 3/16 **0.101**	15/16 **0.076**	5/8 **0.050**	7/16 **0.038**	3/8 **0.030**	5/16 **0.025**	1/4 **0.022**	1/4 **0.019**	3/16 **0.017**	3/16 **0.015**	1/8 **0.010**	1/16 **0.007**	1/16 **0.005**		
20	6 7/16 **0.538**	5 15/16 **0.495**	4 3/16 **0.359**	3 1/4 **0.269**	2 3/16 **0.180**	1 5/8 **0.135**	7/8 **0.090**	13/16 **0.067**	5/8 **0.054**	9/16 **0.049**	7/16 **0.038**	7/16 **0.034**	3/8 **0.030**	5/16 **0.027**	1/4 **0.018**	3/16 **0.013**	1/8 **0.009**	1/16 **0.005**	
25	10 1/8 **0.842**	8 7/16 **0.701**	6 3/4 **0.562**	5 1/16 **0.421**	3 1/4 **0.271**	2 1/2 **0.211**	1 11/16 **0.140**	1 1/4 **0.105**	1 **0.084**	11/16 **0.070**	3/4 **0.060**	5/8 **0.053**	9/16 **0.047**	1/2 **0.042**	5/16 **0.027**	1/4 **0.021**	3/16 **0.014**	1/8 **0.008**	1/16 **0.004**
30				7 1/4 **0.606**	4 7/8 **0.405**	3 5/8 **0.303**	2 3/16 **0.202**	1 13/16 **0.151**	1 7/16 **0.121**	1 1/4 **0.101**	1 1/16 **0.087**	15/16 **0.076**	13/16 **0.067**	3/4 **0.061**	1/2 **0.040**	3/8 **0.030**	1/4 **0.020**	1/8 **0.012**	1/16 **0.062**
35					6 5/8 **0.550**	4 15/16 **0.413**	3 5/16 **0.275**	2 1/2 **0.206**	2 **0.165**	1 5/8 **0.137**	1 7/16 **0.118**	1 1/4 **0.103**	1 1/8 **0.092**	1 **0.082**	11/16 **0.055**	1/2 **0.041**	5/16 **0.027**	3/16 **0.016**	1/16 **0.082**
40						6 5/16 **0.538**	4 5/16 **0.359**	3 1/4 **0.270**	2 9/16 **0.216**	2 3/16 **0.180**	1 7/8 **0.154**	1 5/8 **0.135**	1 7/16 **0.120**	1 5/16 **0.108**	7/8 **0.072**	5/8 **0.054**	7/16 **0.036**	1/4 **0.022**	1/8 **0.011**
45						8 3/16 **0.682**	5 7/16 **0.454**	4 1/16 **0.340**	3 1/4 **0.273**	2 3/4 **0.227**	2 5/16 **0.195**	2 1/16 **0.170**	1 13/16 **0.151**	1 5/8 **0.136**	1 1/4 **0.091**	13/16 **0.068**	9/16 **0.045**	5/16 **0.027**	1/8 **0.0135**
50							6 3/4 **0.562**	5 1/16 **0.421**	4 1/16 **0.337**	3 3/8 **0.281**	2 7/8 **0.241**	2 1/2 **0.211**	2 1/4 **0.187**	2 **0.168**	1 5/16 **0.112**	1 **0.084**	11/16 **0.056**	7/16 **0.034**	3/16 **0.017**
60								7 1/4 **0.602**	5 3/4 **0.482**	4 13/16 **0.402**	4 1/8 **0.345**	3 5/8 **0.302**	3 3/16 **0.268**	2 7/8 **0.241**	1 15/16 **0.161**	1 7/16 **0.121**	15/16 **0.080**	9/16 **0.048**	1/4 **0.028**
70									7 7/8 **0.656**	6 9/16 **0.547**	5 5/8 **0.468**	4 15/16 **0.411**	4 3/8 **0.365**	3 15/16 **0.328**	2 5/8 **0.219**	2 **0.164**	1 5/16 **0.109**	13/16 **0.066**	3/8 **0.033**
80										8 9/16 **0.715**	7 3/8 **0.614**	6 7/16 **0.537**	5 3/4 **0.477**	5 1/8 **0.429**	3 7/16 **0.286**	2 9/16 **0.215**	1 3/4 **0.143**	1 **0.086**	1/2 **0.043**
90											9 5/16 **0.777**	8 1/8 **0.679**	7 1/4 **0.603**	6 1/2 **0.543**	4 5/16 **0.362**	3 1/4 **0.271**	2 3/16 **0.181**	1 5/16 **0.109**	11/16 **0.055**
100												10 1/8 **0.838**	8 15/16 **0.744**	8 **0.670**	5 3/8 **0.447**	4 **0.335**	2 11/16 **0.223**	1 5/8 **0.134**	13/16 **0.067**

The superelevation in inches for standard-gage railroad track may be found by multiplying the light-face figures by 4.9, or usually with sufficient accuracy by 5.

Bold-face figures give decimals of a foot, light-face figures in inches.

Because of the effect on slow-moving vehicles, especially when the pavement is slippery, the maximum amount of superelevation must be limited. The consensus of opinion of leading highway engineers is that a value of $e = 0.10$ may be safely used, although values as high as 0.16 may be permitted in special cases. This corresponds closely to the 6 to 10 in. used on high-speed, standard-gage railroads. With the general increase in highway speeds, there is a distinct tendency towards using higher values of e more freely.

It has been common practice to omit the superelevation entirely if the computed value of e is less than about 0.04. From a consideration of safe speeds as given in section 137, and of Moyer's data previously referred to, it appears that, in order to provide a suitable factor of safety against lateral skidding at high speeds on flat curves, a minimum value of e of about 0.02 should be used.

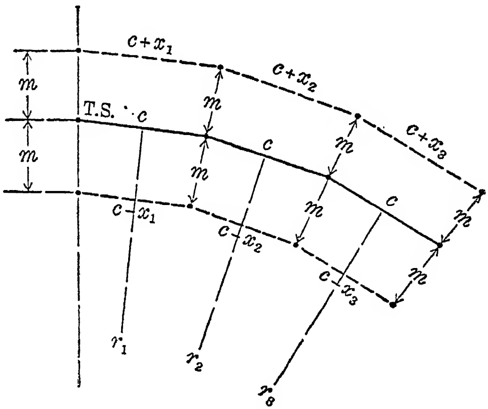


Fig. 43.

158. Spiraling Unwidened Curves. When the pavement width is the same on both tangents and curves, the spirals are best inserted on the center line as outlined in the preceding pages, and the edges of the pavement located from the center-line stakes. This means that offset stakes at a fixed distance from the center line must be located. This is best done by measuring the required offset distance from each center-line stake to the corresponding offset stake. It is essential that the measurements be made *radial* to the curves,

else the alinement of the edges and the width of pavement will be irregular.

Alining by eye. When the offset distance is short, say less than 15 to 20 ft., the offset stakes can usually be lined in with all necessary accuracy by eye.

Using chord and offset. Where the offsets are larger, or greater accuracy is desired, the following method shown in Fig. 43 is convenient: Offset stakes for the inner and the outer edges are set opposite the *T.S.*, using the transit or other accurate method to place them at right angles to the center line of the tangent. The other stakes are then located successively by measuring the offset distance from the corresponding center-line stake and the chord from the preceding offset stake. Since the radius of curvature is different on the offset line from that on the center line, the chord lengths must be changed accordingly. This change in chord length may be computed from the easily derived equation

$$\begin{matrix} cmd & cm \\ 5730 & \end{matrix} \qquad (60)$$

where x is the change in chord length, to be added for the outside edge and subtracted for the inside edge, c is the chord length on the center line, m is the distance from the center line to the line of offset stakes, and r and d are the radius and degree of curve, respectively, of the center line at the middle of the chord c. Values of x can be computed with all necessary accuracy on the slide rule.

On spirals d and r vary from chord to chord, hence x also varies. On the circular curve d and r become D and R, respectively, and x is constant for given values of m and c.

Using transit and tape. Occasionally it may be desired to locate or rerun the offset line with the transit and tape. In this case the same values of a as those used on the center line may be used in conjunction with the chord lengths as given in the preceding paragraph. Although this is not mathematically correct on the spiral, since the chord lengths vary slightly, the discrepancies are exceedingly small and can be neglected. This method, however, is exact on the circular curve. This procedure gives concentric curves with stakes opposite each other, enabling the width of pavement to be checked before the forms are set.

It is, of course, possible to compute a spiral to fit the given values of R, D, and S for each offset line, but this method is cumbersome and adds nothing in the way of accuracy; and, although the curves

are concentric, the stakes are not opposite, making it difficult to check the pavement width before the forms are set.

Some highway departments compute separate spirals for each edge of the pavement, generally using the same length of spiral, or sometimes the same value of k. The alinement so obtained is entirely satisfactory, but the pavement varies somewhat in width along the spiral.

159. Spiraling Widened Curves. As pointed out in section 135, a curve is usually widened by placing the additional width along the

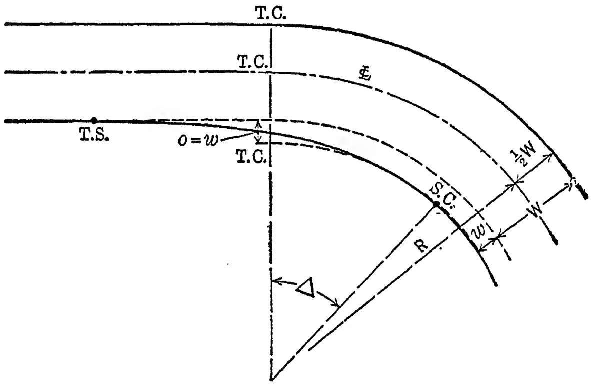


Fig. 44.

inner edge, and also the amount of widening varies with the degree of curve. The spiral is therefore the ideal method of running out the widening, since the width at any point will agree with the degree of curve, and the other advantages of spiraling will also be partially obtained.

Four cases of widened curves may then be considered:

1. Curves widened on the inner edge with both the center line and the outer edge unspiraled.

2. Curves widened on the inner edge with the center line and the outer edge concentrically spiraled.

3. Curves widened on the inner edge with separate spirals on the edges without regard to a center-line spiral.

4. Curves widened symmetrically about the center line with separate spirals on the edges.

160. Case 1. In Fig. 44 the inner edge is widened the amount w and the center line is unspiraled. The outer edge is then a circle concentric with the center line. The inner edge must therefore be spiraled with a curve which must conform to the given radius and width of pavement, and whose value of o must equal the widening w.

From Fig. 44

$$R_1 = R - \tfrac{1}{2}W - w - p$$

where p is the distance from the edge of the pavement to the form stakes.

From Eqs. 30 and 30a,

$$R_1 = \frac{5730}{kS} \quad \text{and} \quad D_1 = kS$$

From Eq. 36, and the given conditions,

$$o = w = 0.0727kS^3$$

Solving for S and reducing,

$$S = \sqrt{\frac{R_1 w}{416.6}} = \sqrt{\frac{w}{0.0727D_1}} \tag{61}$$

With S now known, k is determined from Eq. 30 or 30a.

It is evident that with a given widening and a given radius both k and S will probably have odd values. It is possible, however, to overcome this difficulty by computing S and k as above and then choosing a convenient value of k and computing back for the corresponding value of S and for a new value of $o = w$. The difference between this new value of w and the original value will usually be quite small and may be neglected, or the difference may be adjusted in setting the pavement forms.

Since the alinement will normally be staked out on the center line as a tangent and circular curve, and the stakes for the outer edge located by offsets, it is often convenient to locate the stakes for the inner edge also by offsets. To do this, locate points on the center line at convenient, equal intervals between the $T.S.$ and the $T.C.$,

and also at the same intervals between the $T.C.$ and the $S.C.$ Since the spiral bisects the ordinate o at the $T.C.$, and $o = w$, the offset from the center line at the $T.C.$ to the line stake at the middle of the spiral will be $\frac{1}{2}W + \frac{1}{2}w + p$. The offset from the center line to the $T.S.$ will be $\frac{1}{2}W + p$. The ordinates for the intermediate points can then be found by dividing $\frac{1}{2}w$ in proportion to the cubes of the distances from the $T.S.$ The quantity $\frac{1}{2}W + p$ is added to each of these ordinates to obtain the corresponding offsets from the center-line points. For the other half of the spiral, the offsets are measured from the points on the center-line circle. The offset at the $S.C.$ is $\frac{1}{2}W + w + p$. The intermediate offsets are computed from the same proportional parts of $\frac{1}{2}w$ as for the first part. They are used in the reversed order, working back from the $S.C.$ to the $T.C.$, by subtracting them from $\frac{1}{2}W + w + p$. This will give the same value for the offset at the $T.C.$ as previously computed.

Example. Assume a spiral 200 ft. long and a widening of 2.00 ft. on the inner edge of a pavement which is 20.00 ft. wide on tangents. Form stakes are to be set 2.00 ft. from the edge of the pavement. Divide each half of the spiral into four parts by points 25 ft. apart on the center line, measuring each way from the $T.C.$ The spiral ordinate at the $T.C.$ is $\frac{1}{2}o = \frac{1}{2}w = 1.00$ ft., and at the $T.S.$ it is 0.00. At the first point, 25 ft. from the $T.S.$, the ordinate is $\frac{1.00}{4^3} = 0.0156$, at the second point it is $0.0156 \times 2^3 = 0.12$, and at the third point it is $0.0156 \times 3^3 = 0.42$ ft. Adding each of these to $\frac{1}{2}W + p = 12.00$ ft., the offsets from the $T.S.$ to the $T.C.$ are 12.00, 12.02, 12.12, 12.42, and 13.00 ft., respectively. Now using these same ordinates, and beginning at the $S.C.$ and working back, each is subtracted from $\frac{1}{2}W + w + p = 14.00$ ft., whence the offsets are 14.00, 13.98, 13.88, and 13.58 ft., respectively, with 13.00 ft. at the $T.C.$ as before.

161. Case 2. In this case a suitable spiral is selected for the center line and located in the usual way. The outer edge is then located as a concentric spiral by measuring offsets of $\frac{1}{2}W + p$ from the center-line stakes to the form stakes.

For the inner edge a spiral must be computed for the corresponding radius and for a value of o equal to o for the center-line spiral plus the widening w. The problem therefore becomes identical in form with that in Case 1. Using the subscript 1 for the inner spiral, we have

$$o_1 = o + w \quad \text{and} \quad R_1 = R - \tfrac{1}{2}W - w - p$$

By analogy with Eq. 61,

$$S_1 = \sqrt{\frac{R_1(o + w)}{416.6}} = \sqrt{\frac{o + w}{0.0727D_1}} \tag{61a}$$

S_1 and k_1 are computed as in Case 1, and may be corrected back for an even value of k_1 if desired.

The spiral may be located by offsets in a manner similar to that in Case 1. The center-line tangent is produced through the *T.S.* to the *T.C.* and divided into equal parts as before. The spiral ordinates are computed from $\frac{1}{2}o_1$ and added to $\frac{1}{2}W + p$ to obtain the offsets. For the second half of the spiral it is necessary to extend the center-line circle back from the *S.C.* to the *T.C.* and to locate the required points on it. The offsets from these points are $\frac{1}{2}W + w + p$ minus the spiral ordinates, as before.

162. Case 3. This case differs from Case 2 in that no center-line spiral is considered. A spiral is chosen directly for the outer edge and is located independently. Using the subscript 1 for the inner spiral, to correspond with the preceding cases, and the subscript 2 for the outer spiral, as shown in Fig. 45, we have by analogy,

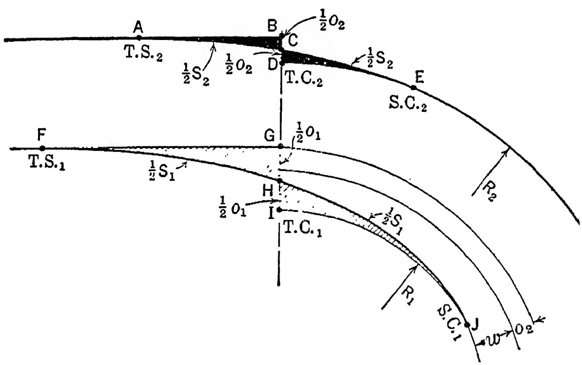


FIG. 45.

$$o_1 = o_2 + w \quad \text{and} \quad R_1 = R - \tfrac{1}{2}W - w - p$$

S_1 and k_1 are determined as before and may be corrected back for an even value of k if desired.

Both spirals may be located by offsets by applying the same principles as used in the previous cases.

163. Symmetrical Widening. Case 4. Satisfactory results can often be obtained by widening the curve symmetrically about the center line with the center line spiraled. There are two methods of doing this.

First Method. A spiral for the center line is chosen, computed, and laid out in the usual way. Spirals are then computed for the outer and inner edges to agree in a manner analogous to the preceding cases.

For the outer edge,

$$o_2 = o - \tfrac{1}{2}w \quad \text{and} \quad R_2 = R + \tfrac{1}{2}W + \tfrac{1}{2}w + p$$

For the inner edge,

$$o_1 = o + \tfrac{1}{2}w \quad \text{and} \quad R_1 = R - \tfrac{1}{2}W - \tfrac{1}{2}w - p$$

The equations for S_2 and S_1 are analogous to Eq. 61a, from which values of S_2, k_2, S_1, and k_1 can be computed. These may be corrected back for even values of k_1 and k_2.

Second Method. The center-line spiral is provided as before. The widening is then made in successive increments by dividing $\frac{1}{2}w$ into ordinates for equally spaced points on the spiral in proportion to the cubes of the distances from the *T.S.* These ordinates added to $\frac{1}{2}W + p$ give the offsets to be measured from the proper points on the center line to both the inner and outer lines of form stakes. This method is simple and easily applied and gives results approximating those obtained by the first method.

It is to be especially noted that if $o = \frac{1}{2}w$ there will be no spiral on the outer edge, and the problem reduces itself to Case 1. If o is less than $\frac{1}{2}w$, the outside widening, if brought around to the *T.C.*, would lie outside the tangent. This is undesirable and therefore symmetrical widening should not be used.

164. Example. Given: center radius $R = 500$ ft., pavement width 20 ft., stakes to be placed 2.0 ft. outside of edge of pavement.

a. Pavement to be widened 2.0 ft. on inside edge with outer edge unspiraled. Then $o_1 = 2.0$ and R_1 (radius of line of stakes) $= 486.0$ ft.

From Eq. 61,

$$S = \sqrt{\frac{486 \times 2}{[illegible]}} = 1.527$$

From Eq. 30a,

$$\frac{5730}{486 \times 1.526} = 7.72$$

Taking $k = 7.7$ and solving back for S and w, we find $S = 1.532$ and $o = w = 2.01$. The difference in width is negligible, and hence the simple value of k would be used.

b. The pavement is to be widened 2.0 ft. on the inside, with the outside edge spiraled with a spiral about 100 ft. long. Then $R_2 = 512$ and $S_2 = 1.0$, whence from Eqs. 30a and 36, $k_2 = 11.2$ and $o_2 = 0.81$, which values are satisfactory.

Then,

$$o_1 = 2.00 + 0.81 = 2.81 \quad \text{and} \quad R_1 = 486$$

From Eq. 61a,

$$S_1 = \sqrt{\frac{486 \times 2.81}{416.6}} = 1.811$$

From Eq. 30a,

$$k = \frac{5730}{486 \times 1.811} = 6.51 \quad \text{(Use 6.5)}$$

c. The pavement is to be widened 1.0 ft. on each side of the center line, and the center line is to be spiraled with $k = 7.0$. Solving the equations, we find $o = 2.23$ and $S = 163.7$, whence $o_1 = 3.23$ and $o_2 = 1.23$.

Also $R_1 = 487$ and $R_2 = 513$, whence

$$S_1 = \sqrt{\frac{487 \times 3.23}{416.6}} = 1.943$$

$$S_2 = \sqrt{\frac{513 \times 1.23}{416.6}} = 1.230$$

$$k_1 = \frac{5730}{487 \times 1.943} = 6.06$$

$$k_2 = \frac{5730}{513 \times 1.230} = 9.08$$

165. Computations and Field Work. The computations and field work for highway spirals are identical with those for railroad spirals as previously outlined. It should be remembered, however, that, if spirals at the edges of a pavement are run separately by transit and tape, they should be computed for the *radius at the stake line* and not for the radius at the edge of the pavement or that of the center lines.

166. Compound Curves. As pointed out in Chapter 4, compound curves are very undesirable on highways from the standpoint of

traffic, since the unexpected change in curvature introduces hazards. If, however, the topography demands a compound curve and the difference between the two degrees of curve is more than about 1°, the road will be made safer by the introduction of a spiral of generous dimensions between the two circles. Such spirals are computed and laid out in the same manner as previously explained.

167. Area of Pavement due to Spiraling. Since the spiral departs from any osculating circle at the same rate as from the initial tangent, and since o and S mutually bisect, it follows that the area between the initial tangent, the first half of the spiral, and the upper half of the ordinate o is equal to the area between the second half of the spiral, the lower half of the ordinate o, and the circular curve from *T.C.* to *S.C.* Thus in Fig. 45 the area ABC equals the area CDE, and likewise the area FGH equals the area HIJ.

It is therefore evident that the insertion of a spiral does not of itself affect the area of the pavement. The area of pavement on a widened curve, when spirals are used, is therefore equal to the area found by multiplying the total width of the widened portion by the length of the circular arc whose radius is equal to $\frac{1}{2}(R_1 + R_2)$ and whose central angle is the intersection angle I of the given curve. In other words, compute the area of pavement as if it were to be widened abruptly at the *T.C.* and *C.T.* without any kind of transition.

Problems

1. What is the maximum speed for 10°, 7°, 3°, and 2° curves?

2. Given. *P.I.* at Sta. 741 + 60.0; $I = 35°\ 42'$; $k = 2°$; and $D = 5°\ 00'$. Determine the station numbers of the *T.S.*, *S.C.*, *C.S.*, and *S.T.*, and the external distance of the curve.

Answer. *T.S.* = 736 + 65.3, *S.C.* = 739 + 15.3, *C.S.* = 743 + 79.3, *S.T.* = 746 + 29.3, $E_s = 60.3$.

3. Write transit notes for Problem 2.

4. Compute notes for locating the spiral in Problem 2 by offsets.

5. On new location a 5° 00′ curve is to be offsetted from a 2° 00′ curve at Sta. 333 + 00.0 for a $k = 1°$ spiral. Find the station numbers of the *C.S.* and the *S.C.* and the required offset. Write notes for spiral.

6. Write notes to locate spiral in Problem 5 by offsets.

7. Given. *P.I.* at Sta. 267 + 0.0; $I = 69°\ 20'$; $I_1 = 21°\ 40'$; $I_2 = 47°\ 40'$; $D_1 = 1°\ 30'$; $D_2 = 4°\ 00'$; $k_1 = k_2 = 1°$; $k_3 = 2°$.

Find T_{s1}, T_{s2}, and the station numbers of the $T.S.$, $S.C._1$, $C.S._1$, $S.C._2$, $C.S._2$, and $S.T.$

Answer. $T_{s1} = 1883.8$, $T_{s2} = 1271.9$, $T.S. = 248 + 16.2$, $S.C._1 = 249 + 66.2$, $C.S._1 = 262 + 10.6$, $S.C._2 = 264 + 60.6$, $C.S._2 = 274 + 27.3$, $S.T. = 276 + 27.3$.

8. Write transit notes for Problem 7.

9. An existing curve is to be spiraled. $I = 39° 18'$, $D = 3° 20'$, $k = 1°$. The track is to be shifted outward at the middle about 1.0 ft. Find D_1 and the actual value of p.

Answer. $D_1 = 3° 28'$; $p = 0.86$ ft.

10. Given an existing 2° 40′ curve to be spiraled by compounding near the ends with a 3° 00′ curve. The $T.C.$ is at Sta. 66 + 66.6 and $k = 1°$. Find the station numbers of the $T.S.$, $S.C.$, and $C.C.$, and the amount that the track will be shortened.

Answer. $T.S. = 65 + 47.2$, $S.C. = 68 + 47.2$, $C.C. = 69 + 42.2$. The track is shortened 0.06 ft.

11. An existing compound curve has its $C.C.$ at Sta. 488 + 50.0. $D_1 = 4° 00'$, $D_2 = 7° 00'$, $D_3 = 7° 40'$, and $k = 1°$. Find the station numbers of the $C.S.$, $S.C.$, and $C.C.$

Answer. $C.S. = 487 + 16.5$, $S.C. = 490 + 83.2$, $C.C. = 491 + 24.2$, $n_1 = 49.87$, $n_2 = 224.40$.

12. A highway curve has a normal center radius of 400 ft. and a normal pavement width of 18.0 ft. The curve is to be widened 3.0 ft. with form stakes placed 2.0 ft. from the edge of the slab. With the outer edge unspiraled, find the value of S and k for a spiral on the stake line of the inner edge.

Answer. $S = 1.667$; $k = 8.9$.

13. The curve in Problem 12 is to have a spiral about 100 ft. long on the outer edge. Find the computed value of k_2, choose an even value, find the corresponding value of S_2, and o_2, and then compute the values of S_1 and k_1.

Answer. k_2 computed $= 13.94$; k_2 used $= 14.0$; corrected $S_2 = 0.996$; $S_1 = 1.927$; $k_1 = 7.67$; $o_1 = 4.01$; $o_2 = 1.01$.

14. The curve in Problem 12 is to be widened symmetrically about the center line, using $k = 10.0$ for the center-line spiral. Find o, o_1, o_2, S_1, S_2, k_1, and k_2.

Answer. $o = 2.14$; $o_1 = 3.64$; $o_2 = 0.64$; $S_1 = 1.840$; $S_2 = 0.796$; $k_1 = 8.04$; and $k_2 = 17.45$.

CHAPTER 6

STRING-LINING RAILROAD CURVES

Curved track gets out of alinement much faster than straight track. This is due to large and variable lateral forces, principally unbalanced centrifugal force, resulting from the absence of spirals, variations in the superelevation, and the fact that few trains run at the theoretical speed for which the track is superelevated. The action of these forces is hastened by lack of uniform support by the ballast and subgrade, which permits the track to weave up and down. Consequently, curves must be realined frequently in order to be safe and smooth riding, especially at high speeds.

If the curve data are known and the control points can be accurately relocated, the curve may be rerun with the transit and tape as in the original location. Or, if the data are not known and the points can not be relocated, the transit may be set up at a point on the curve and deflection angles read to other points, from which data an approximate curve can be worked out and the curve run-in with transit and tape. This is known as the *deflection-angle method* of realinement. It gives good results but may require excessive shifting of the track, and the process is subject to serious delays if train movements are frequent.

String-lining is a simpler process based on the principle of middle ordinates, which does not require the use of the transit and tape. It *does not* reestablish the original curve. Instead, it substitutes for it a curve whose length is some multiple of the chosen station spacing and whose curvature may vary slightly from point to point, but which will provide a safe and smooth-riding track with the minimum of shifting. Both the equipment and the field work are very simple, and the process is subject to a minimum of interruption by train movements. Because of its advantages this method has become standard practice on many railroads.

168. Stationing. The string to be used must be the same length as the chosen chord. Any length of chord may be used, but in general only two lengths are commonly employed. The American

Railway Engineering Association recommends a 62-ft. chord principally because with this length the middle ordinate to it, in inches, is practically equal to the degree of curve. Some engineers prefer to use a 78-ft. chord, or twice the standard rail length, since with it the rail joints may be used as the stations. Chords shorter than 62 ft. are often used on very sharp curves, especially above about 15°, because stakes are usually desired at shorter intervals and also, with the longer chords, the middle ordinates become too large for convenient use.

Since the middle ordinates are measured at the middle of the chords, the *stations* are spaced one-half the chord length apart. Hence the total length of the string-lined curve must be some multiple of the half-chord length. The stations are numbered consecutively, starting with 0, and the work can begin at either end of the curve irrespective of any previous stationing.

169. Locating the Stations. The first step is to locate the beginning point, *T.S.* or *T.C.* With the 78-ft. chord this is always taken at a rail joint, but with the 62-ft. or shorter chords it is located without respect to the rail joints.

If the curve data are known and the original beginning and end points of the curve can be relocated, the exact position of these points for the string-lined curve can be determined by considering the length of the original curve in relation to some multiple of the half-chord length. Since with the 78-ft. chord the stations are at rail joints the string-lined curve may not be symmetrical with respect to the original curve, but this is unimportant.

If the curve data are not known and the original beginning and end points can not be relocated, the starting point for the string-lining is determined by eye, perhaps with the assistance of stretching a string to determine where an ordinate first appears. In fact, this is the usual method in any case. The point so chosen is the *T.S.* or *T.C.* of the string-lined curve. The corresponding point at the far end of the curve is the *S.T.* or *C.T.* and is located at the nearest string-lining station, depending on the chord length used.

Since there will be an ordinate at the beginning point and also at the end point of the curve, to a string stretched between a station on the curve and one on the tangent, it follows that there *must* be at least one station on the tangent back of the beginning point and also beyond the end point. But, since these points are somewhat indefinite and may be shifted, it is well to have several stations on the tangent. If the tangent is in good alinement or will be made

so as a separate operation, two or three extra stations are enough, but if part of the tangent is to be alined more should be included. Therefore, Sta. 0 is set several stations back of the *T.C.* or *T.S.*, as indicated in Fig. 46, and the station of the beginning point recorded. At the far end the station of the *C.T.* or *S.T.* is recorded, and the stations are continued well beyond that point.

After Sta. 0 is located, the other stations are set from it by using the rail joints or measuring the distances. All this work is done on the *gage* side of the *outer* rail. With the 78-ft. chord no actual mea-

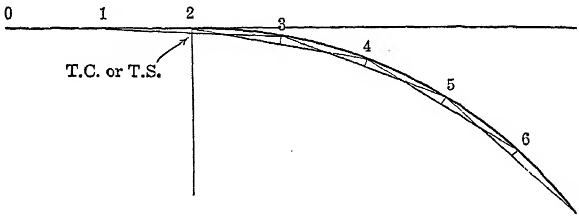


FIG. 46.

surements are made, since the rail joints are used. The stations, however, should not be at the actual joint because of small irregularities in the gage line at the rail ends. They should be set a short distance from joint, the end of the splice bar being a convenient and definite point to use. With the 62-ft. and shorter chords the measurements are made on top of the rail. The point and station number should be marked on the web or flange of the rail with keel.

Three men are required to take the data. Two stretch the string, while the third measures and records the ordinates. If the rail joints are used, the leading stringman can also mark the stations; hence the whole field process can be done in one operation. With the 62-ft. chord two men are required to locate the stations and three to measure the ordinates in separate operations, but the work can be done in one operation by using four men. The first holds the rear of the string, the second holds the head of the string and the rear of the tape, the third holds the head of the tape and marks the stations, and the fourth measures the ordinates.

170. Middle Ordinates. Any convenient unit may be used for measuring the middle ordinates. For rough work the unit may be as large as $\frac{1}{4}$ in., but on average work the unit is usually $\frac{1}{8}$ or $\frac{1}{10}$ in.

For precise work on high-speed tracks $\frac{1}{16}$ or $\frac{1}{20}$ in. is preferable. With the 78-ft. chord the usual unit is 0.01 ft., but $\frac{1}{8}$ in., which is approximately the same, may be used if a decimal scale is not available.

The field work is greatly facilitated by making or purchasing two paddles, shown diagrammatically in Fig. 47. The string passes

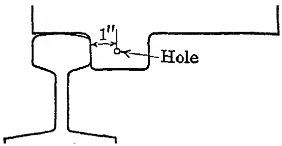


FIG. 47.

through the holes in the paddles, which are held against the rail head as shown. Since the gage line is $\frac{5}{8}$ in. below the top of the rail, the paddles should bear against the rail at this point, and the holes should be at the same elevation. With the holes 1 in. from the gage line, the string is parallel to, but 1 in. away from, the actual chord to which measurements must be made; hence this quantity must be subtracted from measurements made to the string.

The ordinates are measured continuously in units and not in inches and fractions; hence the scale should be so numbered. If the paddles described above are used, the zero of the graduations should begin 1 in. from the end of the rule to eliminate making arithmetical subtractions. Special string-lining scales graduated in this manner, as indicated in Fig. 48, are on the market, or an ordinary scale may

FIG. 48.

be easily renumbered in the same manner with waterproof drawing ink or paint.

171. Sum of Middle Ordinates. A fundamental principle of string-lining is that, for a given total change of direction or central angle, I, and a constant length of chord, the sum of all the middle ordinates on the curve is a fixed quantity. It makes no difference as to the degree of curve, the nature of any spirals, or the extent of irregularity of the curve. The sum of the ordinates must be a con-

stant; hence the sum of the ordinates after string-lining must equal the sum of the ordinates as measured in the field.

The *old ordinates*, as measured in the field, will vary in value because the curve is out of alinement. To be realined the track must be shifted, inward here and outward there, until all the ordinates on the circular curve have practically the same value, while those on the spirals increase or diminish with a practically constant increment, and the sum of these *new ordinates* is the same as the sum of the old ordinates.

The distance the track is shifted laterally at any point is called the *throw*. If the throw is *outward*, it is a *plus* throw; if *inward*, it is a *minus* throw.

172. Effect of a Throw. The effect on the middle ordinates of a throw at one station is illustrated in Fig. 49. A, B, C, D, and E

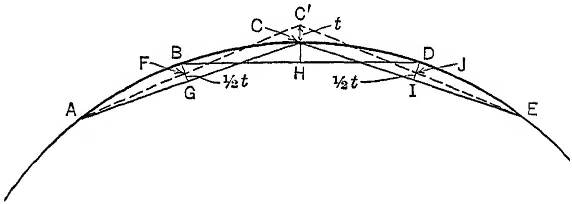


Fig. 49.

are stations on an existing curve. The middle ordinates at B, C, and D are BG, CH, and DI, respectively. If the track is thrown outward at C to C', it is obvious that the ordinate at C has been increased by the amount of the throw t. In making this throw the chord AC would be shifted to AC' and the chord CE moved to $C'E$. Since B and D are the middle points of these two chords, and, since FG, CC', and DJ are practically parallel (the figure is greatly exaggerated), the ordinates at these points are $\frac{1}{2}t$, since $FG = JI = \frac{1}{2}CC'$.

From the foregoing, two basic principles may be stated as follows:

1. An *outward* or plus throw at any station *increases* the ordinate at that station by the amount of the throw and *decreases* the ordinate at the two adjacent stations by *one-half* the throw.

2. An *inward* or minus throw at any station *decreases* the ordinate at that station by the amount of the throw and *increases* the ordinate at the two adjacent stations by *one-half* the throw.

From these principles it is evident that a throw does not change the sum of the ordinates, since a change of the ordinate due to the throw at the given station is balanced by changes in the ordinates at the two adjacent stations in the opposite direction, amounting to one-half the throw.

173. New Ordinates. A series of new ordinates must be found which will satisfy the following requirements:

1. The sum of the new ordinates must exactly equal the sum of the old ordinates.
2. The ordinates at all points on the circular curve must be *practically* equal (sections 117 and 174).
3. The ordinates at successive points on the spirals must increase or diminish at a *practically* constant rate (sections 138 and 174).
4. The resulting throws must not require the track to be shifted at any point more than a chosen maximum, usually less than 8 in.

Theoretically, from the first three requirements, there should be but one fixed answer to the problem. Practically, however, in view of the fourth requirement, there may be several answers, differing somewhat but each satisfactory in providing a smooth-riding and safe track. The attempt is always made to get a solution which results in a minimum amount of throw.

174. Variations in New Ordinates. Since both the old and the new ordinates are taken only to whole units, which means that the throws are taken to two whole units, and, since the degree of curve or the nature of the spirals may theoretically call for fractional units, it will rarely happen that a series of new ordinates can be found which will exactly meet the requirements in section 173, except as to their total sum. Consequently, it is necessary to permit small variations in the new ordinates or spiral increments.

The effect of a small difference in the ordinates at adjacent points is to change the degree of curve slightly. With the 62-ft. chord, units of $\frac{1}{20}$, $\frac{1}{16}$, $\frac{1}{10}$, $\frac{1}{8}$, and $\frac{1}{4}$ in. are equivalent to changes in the degree of curve of 3, $3\frac{3}{4}$, 6, $7\frac{1}{2}$, and 15 minutes, respectively. With the 78-ft. chord a unit of 0.01 ft. is equivalent to about $4\frac{1}{2}$ minutes in the degree of curve. Therefore, if adjacent ordinates do not differ more than one unit, the slight change in the degree of curve is inappreciable and is always neglected.

Another factor that may influence the disposition of variations in the new ordinates is the desirability of having an approximate balance between the plus and minus throws. If the inward throws are excessive, the rails may be unduly crowded together at the joints, or with excessive outward throws the rails may be separated an undesirable amount.

175. Procedure. On a circular curve the simplest method of finding a trial value for the new ordinates is to average the old ordinates. If the curve is unspiraled, the sum of the old ordinates is divided by the number of chords on the curve. If the curve is spiraled, the ordinates between, but not including, the *S.C.* and *C.S.* are averaged.

It will rarely happen that such an average will be a whole number. Consequently variations in the new ordinates to absorb the fractions will have to be made in order that the sum of the new ordinates will equal the sum of the old ordinates.

On an unspiraled curve one-half of the trial ordinate to the nearest whole unit becomes the new ordinate at the *T.C.* and *C.T.*, since they are measured to a chord one-half of which is on the tangent and one-half on the curve. The trial ordinate may then be written opposite the other stations with enough of them increased or diminished by one unit to give the required sum. These are still only trial ordinates, since the variations may have to be redistributed in the process of determining the throws.

On a spiraled curve, tentative values for the circular portion may be worked out in this manner, subject to adjustment first because of fractional units in the spiral increments, and later for the redistribution of the variations.

176. New Spiral Ordinates. From Eq. 44, section 144:

$$m \text{ (feet)} = 1.09n^2(s_1 + s_2)k$$

For the 62-ft. chord, $n = 0.62$, whence

$$m \text{ (feet)} = 0.042(s_1 + s_2)k$$

Reducing m from feet to units of a fraction of an inch,

$$m \text{ (units)} = \frac{0.5}{u}(s_1 + s_2)k \qquad (62)$$

For the 78-ft. chord and a unit of 0.01 ft.:

$$m \text{ (units)} = 66(s_1 + s_2)k \qquad (62a)$$

where u is the value of the unit as a fraction of an inch.

Applying Eq. 62 to a spiral with its *T.S.* at Sta. 3:

At Sta. 4, for the chord from Sta. 3 to Sta. 5,

$$m_1 = \frac{0.5}{u}(0.00 + 0.62)k = \frac{0.31}{u}k$$

At Sta. 5, for the chord from Sta. 4 to Sta. 6,

$$m_2 = \frac{0.5}{u}(0.31 + 0.93)k = \frac{0.62}{u}k = m_1 + \frac{0.31}{u}k$$

At Sta. 6, for the chord from Sta. 5 to Sta. 7,

$$m_3 = \frac{0.5}{u}(0.62 + 1.24)k = \frac{0.93}{u}k = m_2 + \frac{0.31}{u}k$$

From these values it is evident that the term $\frac{0.31}{u}k$ is not only the ordinate at the first station on the spiral but is also the *increment* to be added to the ordinate at one station to obtain the ordinate at the next.

Eq. 62 is rarely used in practice. The simplest way to determine the increment is to divide the trial ordinate for the circle by the number of chords on the spiral. This is not likely to be a whole number, hence in computing the ordinates variations may be needed in the increments to compensate for the fractions.

Although there is no throw at the *T.S.* or *S.T.* there must be an ordinate since one end of the string is on the tangent and the other on the spiral. Table 5 shows that the ordinate at the *T.S.* is practically one-sixth of the increment. Therefore, at the *T.S.* and *S.T.* a new ordinate equal to one-sixth of the increment to the nearest unit may be adopted.

At the *S.C.* or *C.S.* the string will be stretched between a point on the spiral and one on the circle; hence the ordinate will be less than if the chord were entirely on the spiral. Since one-sixth of an increment was used at the *T.S.*, the remaining five-sixths becomes the increment to the *S.C.* If a full increment were added, the result should be the trial ordinate for the circle; hence the ordinate at the *C.S.* should also be equal to the circular ordinate minus the ordinate used at the *T.S.*

TABLE 5

MIDDLE ORDINATES FOR SPIRALS, BASED ON 62-FT. CHORDS

Sta.	Middle Ordinates in 0.05-in. Units									
	Increment = 3 units, $k=0.48$	Increment = 4 units, $k=0.64$	Increment = 5 units, $k=0.80$	Increment = 6 units, $k=0.96$	Increment = 7 units, $k=1.12$	Increment = 8 units, $k=1.28$	Increment = 9 units, $k=1.44$	Increment = 10 units, $k=1.60$	Increment = 11 units, $k=1.76$	Increment = 12 units, $k=1.92$
0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1	$1\frac{1}{6}$	$1\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{2}{3}$	$1\frac{5}{6}$	2
1	3	4	5	6	7	8	9	10	11	12
2	6	8	10	12	14	16	18	20	22	24
3	9	12	15	18	21	24	27	30	33	36
4	12	16	20	24	28	32	36	40	44	48
5	15	20	25	30	35	40	45	50	55	60
6	18	24	30	36	42	48	54	60	66	72
7	21	28	35	42	49	56	63	70	77	84
8	24	32	40	48	56	64	72	80	88	96
9	27	36	45	54	63	72	81	90	99	108
10	30	40	50	60	70	80	90	100	110	120
11	33	44	55	66	77	88	99	110	121	132
12	36	48	60	72	84	96	108	120	132	144
13	39	52	65	78	91	104	117	130	143	156
14	42	56	70	84	98	112	126	140	154	168

With the ordinates for the spirals determined, the final adjustment of the ordinates on the entire curve to give the required sum can be made.

The next step is to determine the throws. Two methods are in common use. These will be treated in sections 177 and 178.

177. A.R.E.A. Method. Although this method of determining the throws is, like all others, based on the principles of the effect of a throw as given in section 172, the application of the principles is made in an indirect manner not readily apparent. The process determines the throws consecutively from one end of the curve to the other. Consequently a check can not be made or the final adjustment of the ordinates be completed until the entire curve has been considered.

With the stations, the old ordinates, and the new ordinates in adjacent columns, the next step is to determine the *errors*. The error at any station is the amount that the old ordinate must be changed to give the new ordinate; hence it is the difference between the old and new ordinates. It is found by subtracting, algebraically, the new ordinate from the old ordinate, since this will give the correct algebraic sign for the throw required at the next station to eliminate the error.

The next step is to determine the *residual error* at each station. Since the process realines the curve as it proceeds, thus eliminating the errors successively, it follows that the error to be eliminated at any station by a throw at the next station is the algebraic sum of the errors from the beginning to and including the given station. This operation is called the *summation of errors*. The first error with its proper sign is recorded opposite its station in the summation column. It is then added, algebraically, to the error at the next station and recorded opposite that station as the residual error. This residual error is then added to the error at the next station, and so on around the curve. Since the sum of the new and old ordinates must be the same, the final summation of the errors at the *C.T.* or *S.T.* must be *zero*.

The next step is to determine the *half-throw* at each station. To simplify the explanation an unspiraled curve will be considered, but the process is the same on a spiraled curve. At the *T.C.* the chord extends from a point on the tangent which can not be moved to a point on the curve which can be shifted. Therefore, an error at the *T.C.* can be eliminated only by a throw at the first station on

the curve. By the principles of section 172 the amount of this throw would be twice the amount of the error at the *T.C.* Therefore the error at the *T.C.* becomes the half-throw at the first station on the curve. Since the errors were determined in a manner to give the correct algebraic sign for such a throw, the error at the *T.C.* with its proper sign is entered in the half-throw column opposite the first station on the curve.

If the second station on the curve were to retain its relative position with respect to the first station, it would be necessary to throw it the same amount and in the same direction as the first station, or its half-throw would be the same. However, there is a residual error at the first station which, because the curve has been realined to the first station, can be eliminated only by a throw of twice its amount at the second station. Therefore, the half-throw at the second station is the algebraic sum of the half-throw and residual error at the first station. Similarly, the half-throw at the third station is the algebraic sum of the half-throw and the residual error at the second station, and so on around the curve.

As a mechanical aid, it is to be noted that in making the summation of errors the algebraic additions are made downward to the left and then recorded horizontally to the right, while in determining the half-throws the algebraic additions are made horizontally to the left and then recorded downward one line to the right.

Since there can be no throw at the *C.T.*, the half-throws must become zero at this point. It will often happen, however, that there will be an "error of closure" at the *C.T.*; i.e., the half-throws will not become zero. This does not indicate a mistake. It merely means that the variations in the new ordinates have been so distributed that the curve is displaced somewhat laterally and hence does not meet the tangent as required.

In explanation, assume that at some station the new ordinate is not exactly correct. Consequently the degree of curve is not exact, with the result that the tangent at the end of the chord does not have quite the correct direction. Even if the curve were exactly realined beyond that point, it would be displaced slightly laterally an amount which would increase directly with the distance; hence the curve would miss the *C.T.* Obviously this displacement could be corrected by a similar distortion in the opposite direction at any convenient station. This provides the means for eliminating the error of closure by redistributing the variations in the new ordinates.

For example, assume that there is an error of closure of +6. At any convenient point more than 6 stations back along the curve, a new ordinate is changed by one unit in such a manner as to decrease a plus error or increase a minus error. Then, 6 stations later, the new ordinate is changed one unit in the opposite direction to keep the sum constant. Thus, if the first ordinate is increased one unit, the second must be decreased one unit. Between these two stations the residual errors will each be changed by one unit; hence in summing up for the half-throws they will make a total change of 6 units in a manner which will eliminate the error of closure.

If the error of closure is greater than the available number of stations, the ordinates at 2 or more stations can be changed as required and the change-back made for each at such stations that the total number of ordinates affected will equal the number of units in the error of closure.

On a long curve it may be found that the half-throws are becoming too large. This difficulty can often be overcome by applying the process outlined above. At a convenient station back along the curve a new ordinate is changed in such a manner as to reduce the throws. A new summation of errors and half-throws is then made until the throws are within the desired limit, and then the ordinate is changed to keep the sum constant. This may be done at several places on the curve as required. This process also permits adjustments to keep an approximate balance between the total amount of plus and minus throws.

The final step is to compute the throws, which is done by merely multiplying the half-throws by two. The throws may be reduced to inches and fractions or retained in units, depending on the equipment to be used in staking out the curve for realinement.

Example 1 shows a simple case of an unspiraled curve. The sum of the ordinates is 805, and there are 20 stations on the curve. Therefore the approximate value of the new ordinate is 40+. New ordinates of 20 were placed at the *T.C.* and *C.T.*, and it was found that the sum could be kept constant by making five ordinates of 41 and the remainder 40. On a first trial these five ordinates of 41 were placed in a group near the middle of the curve, but the summations showed that the throws were excessive and the error of closure was too large. The group was then shifted as shown under the "Trial" heading, which resulted in an error of closure of +7. This was adjusted out in the final solution, which gives new ordinates

conforming to the requirements with throws of small magnitude and hence is satisfactory.

Station	Pt.	Old Ordinate	Fourth Trial				Final				
			New ordinate	Error	Sum	$\frac{1}{2}t$	New ordinate	Error	Sum	$\frac{1}{2}t$	t
0		0	0	0	0	0	0	0	0	0	0
1		0	0	0	0	0	0	0	0	0	0
2	*T.C.*	18	20	−2	−2	0	20	−2	−2	0	0
3		37	40	−3	−5	−2	40	−3	−5	−2	−4
4		38	40	−2	−7	−7	40	−2	−7	−7	−14
5		46	40	+6	−1	−14	40	+6	−1	−14	−28
6		48	40	+8	+7	−15	40	+8	+7	−15	−30
7		44	41	+3	+10	−8	41	+3	+10	−8	−16
8		37	41	−4	+6	+2	41	−4	+6	+2	+4
9		35	41	−6	0	+8	41	−6	0	+8	+16
10		38	41	−3	−3	+8	41	−3	−3	+8	+16
11		41	41	0	−3	+5	41	0	−3	+5	+10
12		45	40	+5	+2	+2	41 †	+4	+1	+2	+4
13		43	40	+3	+5	+4	40	+3	+4	+3	+6
14		38	40	−2	+3	+9	40	−2	+2	+7	+14
15		35	40	−5	−2	+12	40	−5	−3	+9	+18
16		35	40	−5	−7	+10	40	−5	−8	+6	+12
17		42	40	+2	−5	+3	40	+2	−6	−2	−4
18		46	40	+6	+1	−2	40	+6	0	−8	−16
19		43	40	+3	+4	−1	39 †	+4	+4	−4	−8
20		39	40	−1	+3	+3	40	−1	+3	−1	−2
21		38	40	−2	+1	+6	40	−2	+1	0	0
22	*C.T.*	19	20	−1	0	+7*	20	−1	0	0	0
23		0	0	0			0	0	0	0	0
24		0	0	0			0	0	0	0	0
		805	805	* Error of Closure			805		† Changed		

EXAMPLE 1. Procedure and Adjustment of Error of Closure for A.R.E.A. Method. See Example 5 for the complete solution of this problem by the Portser method.

Example 2 shows the final solution for a spiraled curve, which required about ten trials. It also shows by zigzag lines the manner of making the additions in summing the errors and computing the half-throws.

Sta.		Old Ordinates	New Ordinates	Errors	Sum of Errors	Half-throws	Throws
(1)		(2)	(3)	(4)	(5)	(6)	(7)
	0	0	0	0	0	0	0
T.S.	1	5	1	+ 4	+ 4	0	0
	2	9	6	+ 3	+ 7	+ 4	+ 8
	3	14	12	+ 2	+ 9	+11	+22
	4	16	19	− 3	+ 6	+20	+40
	5	20	25	− 5	+ 1	+26	+52
	6	24	31	− 7	− 6	+27	+54
	7	30	37	− 7	−13	+21	+42
	8	40	44	− 4	−17	+ 8	+16
	9	54	50	+ 4	−13	− 9	−18
	10	61	56	+ 5	− 8	−22	−44
S.C.	11	71	61	+10	+ 2	−30	−60
	12	70	62	+ 8	+10	−28	−56
	13	68	62	+ 6	+16	−18	−36
	14	64	62	+ 2	+18	− 2	− 4
	15	55	62	− 7	+11	+16	+32
	16	53	62	− 9	+ 2	+27	+54
	17	58	62	− 4	− 2	+29	+58
	18	56	62	− 6	− 8	+27	+54
	19	59	62	− 3	−11	+19	+38
	20	68	62	+ 6	− 5	+ 8	+16
	21	64	62	+ 2	− 3	+ 3	+ 6
	22	60	62	− 2	− 5	0	0
	23	58	62	− 4	− 9	− 5	−10
	24	69	62	+ 7	− 2	−14	−28
C.S.	25	65	59	+ 6	+ 4	−16	−32
	26	60	54	+ 6	+10	−12	−24
	27	52	48	+ 4	+14	− 2	− 4
	28	38	42	− 4	+10	+12	+24
	29	32	36	− 4	+ 6	+22	+44
	30	23	30	− 7	− 1	+28	+56
	31	18	24	− 6	− 7	+27	+54
	32	15	18	− 3	−10	+20	+40
	33	15	12	+ 3	− 7	+10	+20
	34	10	6	+ 4	− 3	+ 3	+ 6
S.T.	35	4	1	+ 3	0	0	0
	36	0	0	0	0	0	0

EXAMPLE 2. Final Solution for Spiraled Curve by A.R.E.A. Method.

178. Portser Method.* This method is often called the *bracket method* because in the process groups of adjacent stations are considered simultaneously. It differs from the A.R.E.A. method in that the principles of the effect of throws, as given in section 172, are used directly in the computations, and the operations do not proceed successively around the curve.

In this method the approximate value of the ordinate for the circular curve and the ordinates for the spiral should be computed and recorded, but it is unnecessary to make a complete series of new ordinates whose sum is the same as the old ordinates, as in the A.R.E.A. method. These are useful in recognizing and checking the new ordinates as the computations near completion, while the sum is automatically kept constant by the nature of the operations.

The general procedure is as follows:

By inspection of the old ordinates a series of *throws* is assumed, singly or in groups, for as many stations as may seem desirable. The effects of these throws on the ordinates at the various stations involved are then determined in accordance with the principles of section 172. The combined effects at each station are then applied to the old ordinate to give a tentative value of the new ordinate. In this way a series of revised ordinates is obtained. This trial series of revised ordinates is then used with a new series of trial throws in the same manner, giving a second series of revised ordinates approaching a solution. This procedure is repeated until a satisfactory solution is obtained. The last series of revised ordinates then becomes the final new ordinates while the algebraic sum of the trial throws at each station becomes the final throw for that station.

For the beginner, a convenient worksheet consists of the usual columns for stations, old ordinates, new ordinates, and throws and then groups of four columns for the detailed work in each step, as shown in Example 3. The skilled string-liner, however, does not use this form. Instead he uses a form of the type shown in Example 5. The assumed throws for a given step are recorded in a suitable column under "Trial Throws." The effects of these throws are then worked out *mentally* and applied to the old ordinates or to the last revised ordinates for the respective stations, and these new revised ordinates are then recorded in the "Revised Ordinates" section under the corresponding step number. This greatly reduces

* So named from W. Wallace Portser, retired Division Engineer, Grand Rapids Division, Pennsylvania Railroad, who originated the method.

Old Ordinate	First Bracket				Second Bracket				Third Bracket		Final	
	Throw	Effect: Individual	Effect: Combined	Revised ordinates	Throw	Effect: Individual	Effect: Combined	Ordinate	Throw	Effect	New ordinates	Throws
60				60				60			60	0
61		−1	−1	60				60			60	0
60	+2	+2 −1	+1	61		−1	−1	60			60	+2
59	+2	−1 +2 −1	0	59	+2	+2 −1	+1	60		−1	59	+4
57	+2	−1 +2 −1	0	57	+2	−1 +2 −1	0	57	+2	+2	59	+6
59	+2	−1 +2 −1	0	59	+2	−1 +2	+1	60		−1	59	+4
60	+2	−1 +2	+1	61		−1	−1	60			60	+2
61		−1	−1	60				60			60	0
60				60				60			60	0
537				537				537			537	

Example 3. The Effects and Use of Brackets.

the paper work and expedites the procedure. Example 5 is taken from the actual worksheet of a skilled string-liner.

Since an error (section 177) at the *T.S.* or *T.C.* and the *C.T.* or *S.T.* can be eliminated only by a throw at the adjacent station on the curve, such errors may be used at once to determine the new or-

dinates at the end points of the curve and the final throws at the adjacent stations. This process is identical with the first step in determining the throws by the A.R.E.A. method applied to both ends of the curve.

The process of determining the throws may begin at random at any station, but a skilled string-liner can usually by inspection select suitable starting points. Often the procedure is to select those stations where an old ordinate lies between two others, both of which are either greater or less than the one selected. This kink in the alinement may be partially corrected by the throw at the station selected. For example, assume that three adjacent ordinates are 60, 54, and 59, respectively. A throw of +4 at the middle station, by the principles of section 172, would revise these to 58, 58, and 57, smoothing out the kink.

Throws may be assumed at single stations, as in the preceding paragraph, in such a manner that there is no overlapping of their effects, and several such assumptions may be made simultaneously in the same trial. The entire curve may be worked out in this manner, which may prove desirable for a beginner, although it requires many more trials. The work is expedited, however, and the number of trials reduced by the use of *brackets* and *runs* in addition to these *singles*.

A bracket consists of a series of assumed throws for a group of adjacent stations. Usually, all the throws in a bracket have the same value and sign, but they need not be the same. The effects of a bracket with all throws the same are: to change the ordinates at the adjacent stations outside the bracket by one-half the throw with the opposite sign to the throw, to change the end ordinates in the bracket by one-half the throw with the same sign as the throw, and to leave the other ordinates inside the bracket unchanged. If the throws are not all the same, the effect of the bracket is the same as a combination of a constant bracket and one or more singles.

Example 3 is a very simple case of a portion of a curve illustrating the above-indicated effects of brackets. A skilled string-liner might have foreseen that the first two brackets could have been combined in a single bracket of unequal throws, +2, +4, +4, +4, +2, leaving only the final single, or even that this single might have been included, giving +2, +4, +6, +4, +2, which is actually an ascending-descending run. This indicates how skill and experience play an important part in expediting the work.

A *run* is a series of throws for several successive stations in which

the throws differ by a constant increment. A run may be ascending, with the values increasing, or descending, with the values decreasing, or the two may be combined in either sequence. The effects of an ascending run are: to change the ordinate at the first station back of the run by one-half of the first throw with the opposite sign to the throw; to change the ordinate at the last station in the run by one-half the throw at that station plus, numerically, one-half the increment, with the same sign as the throw; to change the ordinate at the first station beyond the run by one-half of the last throw with the opposite sign; and to leave the other ordinates within

Old ordinates	59	60	60	60	59	59	66	54	60
Throws			−2	−4	−6	−8	−10		
Individual effects		+1	−2 +2	+1 −4 +3	+2 −6 +4	+3 −8 +5	+4 −10	+5	
Combined effects		+1	0	0	0	0	−6	+5	
Revised ordinate	59	61	60	60	59	59	60	59	60

EXAMPLE 4. Characteristics of a Run.

the run unchanged. A decreasing run reverses the order of these changes. A run is useful in reducing a kink where two adjacent ordinates differ by several units. The nature and effects of an ascending run are indicated in Example 4.

Example 5 shows the complete solution by the Portser method of the same curve solved in Example 1 by the A.R.E.A. method. This was taken from the actual worksheet of an experienced stringliner who took about 10 minutes to make the entire solution. Contrast this with Example 1, showing only the final two steps, which were preceded by three other trials and also by the computation of an entire series of new ordinates as a preliminary step. A further comparison of the two solutions shows that they are very similar

Station	Point	Old Ordinate	Revised Ordinates															Trial Throws															Final*	
			Steps ←															→ Steps															New Ordinate	Throw
			1	2, 5	3, 6	4	7, 9	8, 10	11	12	13	14	15	16, 17	18	19	20	1	2, 5	3, 6	4	7, 9	8, 10	11	12	13	14	15	16, 17	18	19	20		
0		0																															0	0
1		0																															0	0
2	T.C.	18												19																			19	0
3		37											42	40															−2				40	−2
4		38									42	45	39	40			41											−10					41	−10
5		46								49	44	40	40			41	40									−8	−6	−8				−2	40	−24
6		48							44	41	41	41	41		42	41	41								−6	−6	−4	−6			−2	−2	41	−26
7		44						39	43	40	40	40	40	42	41	41	41							+8	−6	−4	−2	−4		−2	−2	−2	41	−14
8		37					34	39	39	42	42	43	43	41	41	41	41						+10	+8		−2		−2	−4	−2	−2	−2	41	+4
9		35					41	41	41		42		43	41	41	41	41					+6	+10	+8					−4	−2	−2	−2	41	+14
10		38					35	40	40					42	41	41	41						+10	+8							−2	−2	41	+14
11		41						36	40						41	40	40							+8							−2	−2	40	+4
12		45						44	40							41	40															−2	40	−2
13		43		40	38			39									40						+2										40	+2
14		38	33	39	41		39	40											+6	+4			+2										40	+12
15		35	40	37	39		41	40										+10		+4		+4											40	+18
16		35	40		38		40											+10				+4											40	+14
17		42	37	40	41	42	40																										40	0
18		46		42	42	41													−6	−2	−2												41	−10
19		43		43	40	40													−4	−4	−2												40	−10
20		39		39	41	40													−2		−2												40	−4
21		38		39		40																											40	0
22	C.T.	19																															19	0
23		0																															0	0
24		0																															0	0

EXAMPLE 5. Complete Solution by the Portser Method of the Same Problem as Is Solved in Example 1 by the A.R.E.A. Method.

but not exactly the same, which is typical of string-lining, but the two solutions are equally satisfactory. This example also includes several singles, brackets, and runs. A careful examination of the various steps will show more clearly the effects of each of these and the way in which they may be used in the solution of a problem.

179. Compound Curves. Theoretically, on a compound curve, a flatter curve meets a sharper curve with a common tangent at the *C.C.* To comply with this condition, the ordinate at the *C.C.* must be different from that on either curve, since one end of the string would be on the flatter curve and the other end on the sharper curve. It can be easily shown that the required ordinate would be the average of the ordinates for the two curves. Thus, if the ordinate for one curve is 44 and for the other is 62, the ordinate at the *C.C.* would be 53, while those at the two adjacent stations would be 44 and 62, respectively.

Actually, however, this theoretical condition never exists. Even in laying new track the stiffness of the rails will modify the abrupt change in curvature. On track in service the disalinement caused by traffic, combined with usual maintenance operations, soon results in a longer transition, roughly approximating a spiral. This condition will show up in the old ordinates, especially if there is considerable difference between the two degrees of curve. Consequently, in string-lining, the insertion of a spiral becomes an almost automatic practice.

The procedure for determining the new ordinates for this spiral can be explained best by a simple example. The throws can then be computed by either the Portser or A.R.E.A. method.

Suppose a study of the old ordinates indicates that a uniform ordinate of 51 can be used for the first branch of the curve up to Sta. 29, and a value of 82 can be used for the second branch beyond Sta. 34. The middle spiral would then be five stations in length. Since the purpose of the spiral is to bring about a gradual change of the ordinate from 51 for the first branch to 82 for the second branch, and since this change must be accomplished in five stations, the increment per station must be one-fifth of 31, or approximately 6. Hence the spiral whose increment is 6 will best meet the requirements. The ordinate at Sta. 29, when the string is stretched between Sta. 28 on the circular curve and Sta. 30 on the spiral, will be 51 plus one-sixth the spiral increment, or 52. The ordinates at Stas. 30, 31, 32, and 33 will be 58, 64, 70, and 76, respectively. At Sta. 34, the *S.C.*, the ordinate will be 76 plus five-sixths the spiral

increment, or 81. The difference between the ordinates at Sta. 34 and Sta. 35 is one unit, or one-sixth the spiral increment. The numerical values in this example were specially chosen to illustrate the procedure. Generally, a fractional value of the spiral increment would be required; hence variations in the increment would result.

180. Spiraling Old Curves. String-lining offers a simple and convenient method for inserting spirals at the ends of curves originally not spiraled. The basic requirements are identical with those for spiraling existing curves with the transit and tape, as presented in sections 150, 151, and 152.

Since a spiral replaces a piece of tangent equal to one-half the spiral length, Sta. 0 must be placed far enough back of the *T.C.*, located as previously outlined, to accommodate the added length of curve. Similarly, at the far end the stationing must be carried an ample number of stations beyond the *C.T.* If the tangents are in good alinement, all the old ordinates at these extra stations will be zero.

From the principles of the spiral the *T.C.* and *C.T.* must be offset from the tangent to provide room for the spiral. Therefore, all, or portions, of the circular curve must be sharpened somewhat to provide the necessary offset. It is practically impossible to calculate the amount of sharpening required. Consequently, the process is done by trial, and several solutions may be required before a satisfactory one is found. The case where the entire curve is sharpened will be considered first.

The old ordinates are averaged for a preliminary value of the new circular ordinates. This value is then increased by an amount that is estimated to give the necessary sharpening, thus providing the trial value of the circular ordinate to be used in the following steps.

The next step is to choose the length of the spiral. This may be done by the usual methods of choosing spirals, remembering that the spiral must be some multiple of the half-chord length being used. In general it is desirable but not necessary to make the spiral length an even number of chords, since this will place a station at the old *T.C.* With the spiral length and the trial value of the circular curve known, the spiral increments and ordinates can be worked out and the throws computed as previously outlined.

As indicated in section 151, it is desirable to throw the middle of the curve outward a moderate amount, since this will tend to reduce all throws and also maintain a better balance between the

plus and minus throws. This is also desirable because the general effect of the spirals is to cause inward throws along the spirals and perhaps for some distance beyond. For these reasons the Portser method is superior to the A.R.E.A. method because it deals directly with the throws instead of indirectly through errors.

An estimated plus throw, or a series of such throws, may be chosen near the middle of the curve. The throws for the entire curve are then computed, working both ways towards the ends of the curve. If the resulting ordinates and throws are not satisfactory, the chosen sharpening is probably not suitable. In this case a new circular ordinate, changing the amount of sharpening as seems indicated, is selected, and the entire process repeated. Several such repetitions may be required.

On very long curves the amount of sharpening may be so small as to have little effect on the throws over a considerable portion of the middle part of the curve. In this case the middle portion may be worked out practically as a separate circle without regard to the spirals, and then the end portions computed separately in the manner indicated above. This is analogous to the transit and tape method given in section 152. There is little advantage and some disadvantage in this procedure; hence it is rarely used.

181. Staking the Curve. After a satisfactory series of ordinates and throws has been computed, stakes are set opposite each station and accurately marked by measurements from the gage side of the outer rail. These stakes are so set that the marks will be at a fixed distance, usually 2 ft., from the gage line after the track has been shifted the computed throws at each station. The stakes are placed between the ties so that they will not be moved when the track is shifted. If a station falls on a tie, the stake is placed between the ties on the nearest side but not against the tie. The ballast should be removed to a depth well below the bottom of the tie so that the stakes will not be disturbed by the movement of the tie and also for ease in driving the stakes solidly. The tops should be but little above tie level to avoid being accidentally struck and displaced and also to prevent hazard to the workmen.

Since the throws are computed horizontally, the measurements to set the stakes and then realine the track from them must be made horizontally. With the stakes driven to tie level and the track superelevated there will be several inches difference in the elevations of the stake and the top of rail, which must be allowed for in making the measurements. A chaining pin may be held ver-

tically on the stake and the measuring scale leveled by eye, but the work is expedited and made more accurate by the use of a special string-lining instrument which can be purchased.

This instrument is shown diagrammatically in Fig. 50. It consists of a frame with a sliding extension bar, a transverse sliding pin, and a level tube. The frame carries graduations for several

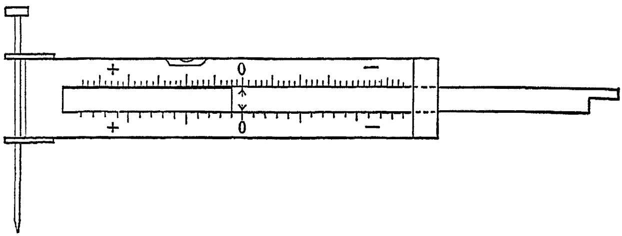


FIG. 50.

different units running both ways from zero for plus or minus throws. For setting the stakes the index point on the sliding bar is set at the desired plus or minus throw, and the shoulder on the bar placed against the rail head. The point of the transverse sliding pin is set on the stake, and the frame slid up or down on the pin until the level bubble is centered. The instrument can also be used for checking the alinement after the throws are made by setting the index point at zero.

CHAPTER 7

PARABOLIC CURVES

PARABOLIC curves are used for vertical curves on grade lines, for pavement crowns, as substitutes for circular curves under certain conditions, and in other cases where it is more convenient to locate the curve by coordinates than by angles and distances.

182. Properties. The equation of the parabola is $y = px^2$. In general the axes are oblique, the X axis coinciding with a tangent and the Y axis paralleling a diameter. From the equation, the following properties can be derived:

1. The offsets from a tangent vary directly with the square of the distance from the point of tangency.
2. A line from the middle of a chord to the intersection of the tangents from the ends of the chord is bisected by the curve. In other words, the external distance E is equal to the middle ordinate M.
3. The second differences between the values of y for equal increments of x are equal.

These fundamental properties make the parabola particularly adaptable to certain work.

183. Compound Parabolas. Occasionally a single symmetrical parabola does not seem to fit the conditions. In the case of horizontal curves this can usually be rectified by using an unsymmetrical curve with unequal tangents. This can not always be done on account of objects interfering with the field work. In this case two or more parabolas can be compounded. The point of compounding is located and the common tangent laid out, and then each branch of the curve is located by the methods given.

On vertical curves, since the coordinates are measured horizontally and vertically, unsymmetrical curves theoretically can not be used. Practically, however, they can be used provided the difference in tangent lengths is not too great, since the effect is merely to distort the curve somewhat and this is of no moment. Occasionally, the best solution lies in using compound curves. The

various tangents are laid out of such lengths that, when the lengths of the vertical curves are fixed, their ends become coincident. Each curve is computed separately.

Vertical Curves

The intersections of the several straight portions of a grade line must be rounded off to avoid undue stresses in drawbars, insure smooth riding, improve the appearance, and provide safe sight distance for automobile drivers. Since the intersection angles are always small, and gravity tends to hold the vehicles in contact with the roadway, the effect of centrifugal force is imperceptible, and the installation of vertical curves is less complicated than that of horizontal curves.

184. Length of Vertical Curve. The length of a vertical curve is dependent on (1) the total change of grade between the two tangents, (2) the safe rate of change of grade per station, (3) the necessary sight distance, and (4) convenience in computing and staking out the curve.

The minimum change of grade which requires a vertical curve is about 0.1 per cent. On roads and streets a greater change than this without a vertical curve will show a distinct break of grade which looks bad, although the natural processes of construction will round the change enough to be acceptable to traffic. On railroads the natural stiffness of the rails will automatically supply a curve of sufficient length.

The tangents of a vertical curve are made equal, and the *P.I.* should be made to come at a regular grade stake. It is convenient, therefore, to make each tangent some multiple of the grade-stake spacing, and hence the total length of vertical curve should be some multiple of twice the grade-stake spacing. On pavements where grade stakes are commonly placed 25 ft. apart, the minimum length of vertical curve becomes 50 ft., and the greater lengths become some multiple of 50 ft. On railroads, grade stakes are normally 100 ft. apart, hence the length of vertical curves should be some multiple of 200 ft., although multiples of 100 ft. are sometimes used.

The maximum length of vertical curve is governed by the topography and the judgment of the engineer. Ordinarily the curves should be made as long as conditions will permit. A grade line with long, easy vertical curves looks better and rides better than one

with short curves on either a highway or a railway. There is a mistaken belief that long vertical curves add to the amount of earthwork. If the grade line is properly designed, both tangents and curves being considered together, the long vertical curves need not add to the earthwork and frequently will actually reduce it.

185. Rate of Change. By definition

$$r = \frac{G_1 - G_2}{L} \tag{63}$$

where r is the rate of change in per cent per station (100 ft.), G_1 and G_2 are the two given grades in per cent, and L is the length of the vertical curve in stations. The grades G_1 and G_2 are considered positive when ascending and negative when descending, and are used with their proper algebraic signs in this equation.

The American Railway Engineering Association recommends $r = 0.1$ for summits and $r = 0.05$ for sags on first-class steam roads. Twice these amounts are recommended on less important lines and on electric roads. Many first-class systems are using the higher rates with apparent satisfaction.

The rate of change is little used in highway work, and then usually in an incidental or auxiliary manner.

186. Sight Distance. Sight distance becomes a controlling factor at summits on highways when the total change of grade is large. On single-lane roads the sight distance should always be something

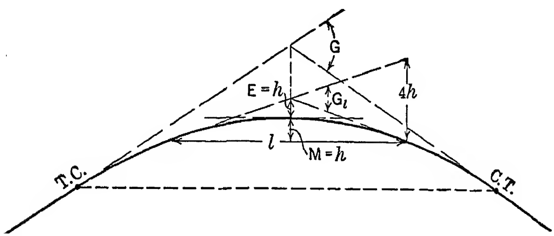


FIG. 51.

more than the sum of the distances in which two vehicles approaching each other can stop. On double-lane roads it is presumed that traffic will stay in the proper lane, and hence the sight distance

should be such that a driver can recognize an approaching vehicle without being startled when traveling at normal road speeds and with the corresponding degree of attention given the road. Experience seems to fix the minimum sight distance on vertical curves at the same value as for horizontal curves, or 700 ft., but with a tendency to use at least 1000 ft. on account of the increasing road

In Fig. 51:

Let L = total length of vertical curve in stations.

l = the sight distance in stations.

h = height of line of sight above the roadway at the point of observation, assumed to be the same for both vehicles, and usually taken as 5.0 ft.

G = total change of grade in per cent.

G_l = change of grade in the sight distance l.

Then from the figure and the properties of the parabola,

$$E = M = h \quad \text{and}$$

$$G_l = \frac{4h}{\frac{1}{2}l} = \frac{8h}{l} \tag{64}$$

By proportion,

$$L = \frac{lG}{G_l}$$

whence

$$L = \frac{l^2G}{8h} \tag{65}$$

Eq. 65 is exact when G is greater than G_l and approximate, but on the safe side, when G is less than G_l.

187. Computing Vertical Curves. There are two common methods of computing vertical curves, each of which has certain advantages under particular conditions. The first method may be called the *tangent offset method* and is generally used on road and street work where the stakes are placed less than a full station of 100 ft. apart. The second method may be called the *rate of change method* and is frequently used on railroads when the stakes are placed at 100-ft. intervals.

188. First Method. In Fig. 52 the elevation of the *P.I.* at A is known, the grades G_1 and G_2 are known, and the length L of the vertical curve has been chosen. Since all distances are theoretically measured on the horizontal, the tangent distances are each equal to $\frac{1}{2}L$, whence the stations of the *T.C.* and *C.T.* at B and C, respectively, can be determined. The elevations of these points can then be found since the grades are known. D, E, F, G, etc., are points where grade stakes are to be placed at equal distances apart, there being n such distances on each side of the *P.I.* Figure 52 is very much exaggerated, as angles ABC and ACB are generally less than 2°.

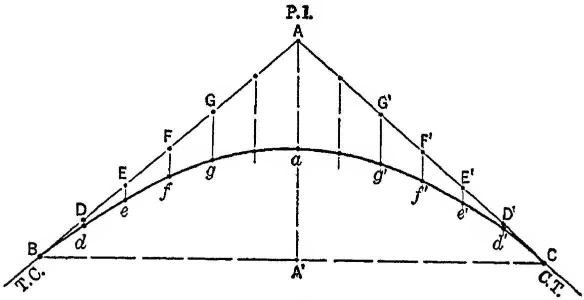


FIG. 52.

The elevations of the points on the tangent at D, E, F, G, etc., are computed, since the distances and grades are known.

The elevation of A', the midpoint of the long chord BC, is readily found, as it is the average of the elevations of B and C.

From the properties of the parabola, $Aa = aA'$, hence the elevation of a and the ordinate Aa can be determined. Also,

$$Dd : Aa :: (BD)^2 : (BA)^2$$

But BD is one unit of n equal units between B and A, hence

$$Dd : Aa :: 1^2 : n^2, \quad \text{or} \quad Dd = \frac{Aa}{n^2} \quad \text{and} \quad Aa = n^2 Dd$$

and

$$Ee = 2^2 \times Dd = 4Dd$$
$$Ff = 3^2 \times Dd = 9Dd$$
$$Gg = 4^2 \times Dd = 16Dd$$

and so on for the number of spaces n.

Having now computed the offsets from the tangent, the elevations of the points on the curve are found by correcting each elevation on the tangent by the corresponding offset. If the curve is at a summit, as shown in Fig. 52, the ordinates are subtracted from the tangent elevations. If the curve is in a sag, the ordinates are found in the same way, but are added to the tangent elevations.

Since the curve is symmetrical about the *P.I.*, it is necessary to compute the offsets for only one-half the curve, since the offsets at the corresponding points on the second half, as at D', E', F', G', etc., are the same as on the first half.

189. Second Method. This method is based on the third property of the parabola as given in section 182. To use this method it is necessary to know, or to be able to find, the station number and elevation of the *P.I.*, the two grades G_1 and G_2, and an approximate value of r.

The first step is to determine the length of curve and the value of r to use. Using the given grades and the approximate value of r in Eq. 63, a trial value of L is computed. This will usually be an odd value which is inconvenient to use and, therefore, the next higher, even number of stations is adopted as the length of curve to use. This value of L is then inserted in Eq. 63 and the value of r to be used is determined. This will generally be an odd value, but it must be used as computed and should be carried out to several decimal places, since any fractions that are dropped will form a cumulative error.

The second step is to determine the station numbers and elevations of the *T.C.* and the *C.T.* which is easily done since the grades, the station number and elevation of the *P.I.*, and the length of curve are known.

The last step is to determine the elevations of the stations on the curve. This is done by first computing the grade of the chord from each station to the next station, and from this new grade determining the elevation of the second station in question.

Since r is the rate of change of grade per station (100 ft.), the difference in grade between successive chords must be equal to r, hence the grade of a chord from one station to the next can be found from the grade of the chord to the preceding station by adding or subtracting r. In these computations r is considered positive, but when the curve is convex upward it is subtracted algebraically from the preceding grade; and when the curve is concave upward it is

added algebraically to the preceding grade. The grades G_1 and G_2 are used with their proper algebraic signs.

Starting at the *T.C.*, the first change of grade is from the tangent to a chord, instead of from a chord to a chord, and is therefore only $\frac{1}{2}r$, hence the grade of the first chord is $G_1 \pm \frac{1}{2}r$. The elevation of the first station on the curve can now be found from the elevation of the *T.C.* The grade of the second chord is $G_1 \pm \frac{1}{2}r \pm r$, and the elevation of the second station is found from the elevation of the first station. The grade of the third chord is $G_1 \pm \frac{1}{2}r \pm r \pm r$, and so on until the *C.T.* is reached. The grade of the chord to the *C.T.*, changed by $\frac{1}{2}r$, should equal G_2, and the elevation of the *C.T.* should check with that found in the second step.

This method can be adapted to other spacings of stakes than 100 ft. by making G_1 and G_2 the slope in feet in the given stake spacing and, at the same time, making r equal the change of slope in feet in the same distance. It is usually preferable, however, to use the first method in such cases.

Example. A $+0.8\%$ grade meets a -0.6% grade at Sta. 30 + 00, whose elevation is 750.50. It is desired to use a vertical curve whose rate of change will be about 0.2% per station.

From Eq. 63,

$$L \text{ (trial)} = \frac{0.8 - (-0.6)}{0.2} = \frac{1.4}{0.2} = 7.000 \text{ Sta.}$$

Making $L = 8.00$ Sta.,

$$r \text{ (to use)} = \frac{1.4}{8.0} = 0.175$$

Taking everything into consideration it is better to make $L =$ 800 ft. (400 ft. on each side of the *P.I.*) and use $r = 0.175$ than it would be to make $L = 700$ ft. and $r = 0.2$.

$$\begin{aligned}
\text{Sta. } T.C. &= 30 + 00 - 4 + 00 = 26 + 00 \\
\text{Sta. } C.T. &= 30 + 00 + 4 + 00 = 34 + 00 \\
\text{Elev. } T.C. &= 750.50 \quad - 4 \times 0.8 = 747.30 \\
\text{Elev. } C.T. &= 750.50 \quad - 4 \times 0.6 = 748.10
\end{aligned}$$

The computations of the chord grades and the elevations of the stations on the curve may be arranged in tabular form as follows:

Station		Computations: Grade	Computations: Elevations	Profile Elevations	Remarks
24			745.7000	745.70	On tangent
			$G_1 = +0.8000$		
25			746.5000	746.50	On tangent
		$+0.8000 = G_1 =$	$+0.8000$		
26	*T.C.*	$-0.0875 = \frac{1}{2}r$	747.3000	747.30	*T.C.*
		$+0.7125$	$+0.7125$		
27		$-0.1750 = r$	748.0125	748.01	
		$+0.5375$	$+0.5375$		
28		$-0.1750 = r$	748.5500	748.55	
		$+0.3625$	$+0.3625$		
29		-0.1750	748.9125	748.91	
		$+0.1875$	$+0.1875$		
30	Vertical Curve L = 800 ft.	-0.1750	749.1000	749.10	
		$+0.0125$	$+0.0125$		
31		-0.1750	749.1125	749.11	
		-0.1625	-0.1625		
32		-0.1750	748.9500	748.95	
		-0.3375	-0.3375		
33		$-0.1750 = r$	748.6125	748.61	
		-0.5125	-0.5125		
34	*C.T.*	$-0.0875 = \frac{1}{2}r$	748.1000	748.10	*C.T.* Check
		-0.6000	$G_2 = -0.6000$		
35			747.5000	747.50	On tangent
			$G_2 = -0.6000$		
36			746.9000	746.90	On tangent

190. Low Point on Vertical Curve. The low point on a vertical curve in a sag must sometimes be located for the purpose of installing storm sewer inlets or other drainage appurtenances.

In Fig. 53, L is the length of the vertical curve, G_1 and G_2 are the

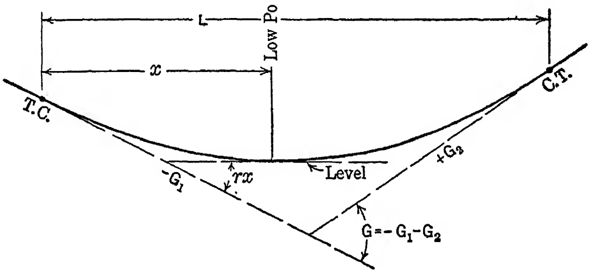


FIG. 53.

two grades, and x is the distance in stations from the $T.C.$ to the low point.

Since the tangent to the curve at the low point must be horizontal, that is, the slope must be zero,

$$G_1 - rx = 0 \quad \text{or} \quad rx = G_1$$

From Eq. 63,

$$G_1 - G_2$$

Substituting this value of r in the preceding expression and reducing,

$$x = \frac{LG_1}{G_1 - G_2} \tag{66}$$

in which G_1 and G_2 must be used with their proper algebraic signs.

Example. $A - 2.0\%$ grade meets a $+ 3.0\%$ grade at Sta. $10 + 00$. A vertical curve 400 ft. long is to be used, and the station of the low point is required. The station of the $T.C.$ is $10 + 00$ minus $2 + 00$, or $8 + 00$.

From Eq. 66,

$$x = \frac{4 \times (-2.0)}{-2.0 - (+3.0)} \quad \frac{-8.0}{-5.0} \quad 1.6$$

The station of the low point is 8 + 00 plus 1 + 60, or 9 + 60. Eq. 66 can also be used to locate the high point on a vertical curve at a summit.

Pavement Crowns

Highway and street pavements are crowned to drain the surface water to the side ditches or the gutters. Normally the crown is convex upward, but on alleys and narrow streets an *inverted crown* with a center gutter is frequently used.

191. Form of Crown. The usual form of crown for roadways of moderate width is a vertical parabola exactly the same as a vertical curve. Some engineers specify an arc of a circle, but the difference between the two is so small that no matter which is specified the crown ordinates can always be computed by the more simple parabolic method.

With widths greater than about 40 ft., a simple parabola may result in cross slopes too steep at the sides, if the middle is crowned enough to drain properly, or the middle is too flat with suitable slopes at the edges. Several forms of crowns have been suggested to overcome these difficulties. The simplest and perhaps best form is a parabola for the middle 35 to 40 ft., flanked by side strips on uniform slopes practically tangent to the parabola.

192. Amount of Crown. The total amount of crown is a function of the width and type of surface, remembering that motor traffic desires as flat a surface as possible. Consequently the average cross slopes should be limited to values between $\frac{1}{16}$ and $\frac{3}{8}$ in. per foot for high type pavements, $\frac{1}{4}$ to $\frac{3}{4}$ in. per foot for gravel and macadam, and $\frac{3}{8}$ to 1 in. per foot for earth roads.

Suitable values of the crown for pavements up to about 40 ft. in width and conforming to present practice may be computed from the equation,

$$C = \frac{W^2}{\quad}$$

where C is the total crown in inches, W is the total width in feet, and m is a factor depending on the kind of surface. For high type pavements m may be taken as 400, for gravel and macadam 200, and for earth 133.

For widths above 40 ft., use values from the above equation for a middle width of 35 to 40 ft. and add uniform slopes on the side

strips of $\frac{3}{8}$ in. per foot for pavements, $\frac{3}{4}$ in. per foot for gravel and stone, and 1 in. per foot for earth. These uniform slopes are essentially tangent to the respective parabolas.

Thus the crown for a concrete pavement 20 ft. wide would be 1 in. For a width of 40 ft. it would be 4 in., and for a width of 60 ft. it would be $4 + 10 \times \frac{3}{8} = 7\frac{3}{4}$ in.

193. Constructing the Crown. In building the crown, the shape both of the subgrade and of the surface is obtained (1) by means of templets cut to the correct shape, (2) by measuring ordinates from a string stretched between the tops of the curbs or grade stakes, or (3) by similar ordinates measured from a line of sight across tees held on the curbs or grade stakes. It is necessary, therefore, to compute the offsets, first from a tangent to the parabola, and then correct these to the desired reference line.

Fig. 54 shows a normal cross-section with the edges of the pavement at the same elevation. A tangent to the parabola at the cen-

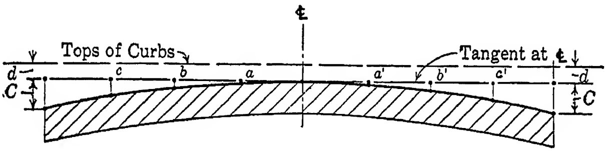


FIG. 54.

ter line passes above the edges of the pavement by the amount of the crown C, and hence C forms a known offset from the tangent at a distance $\frac{1}{2}W$ from the center line. Any other ordinates, as at a, b, c, can then be computed since they must vary as the square of the distance. By making the intervals between the ordinates equal, the various ordinates vary as the square of the digits, as explained under vertical curves. In fact the crown is merely a vertical curve with the tangent at the apex used as an axis. Since the curve is symmetrical, the ordinates at a', b', and c' are the same as at a, b, and c.

The line joining the tops of the curbs will be at a fixed distance d above (or below) the tangent at the center line, hence the ordinates from a line joining the tops of the curbs is equal to the computed ordinates plus (or minus) d. If tees are used, d becomes the distance from the crest of the crown to the line of sight across the tees.

194. Tilted Pavements. On side-hill work it may be desirable to tilt the pavement to save earthwork, reduce curb height, or to improve the appearance. The pavement is also tilted on superelevated curves.

The most common and the simplest method of tilting is merely to set the curbs, or grade stakes, at elevations differing by the amount of the tilting, and then use the regular templet or other

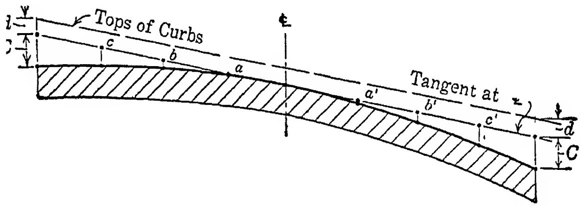


FIG. 55.

usual method of shaping the crown. This condition is illustrated in Fig. 55. The ordinates are the same, and are computed and used in the same way, as in Fig. 54. As the amount of tilting increases, the high point will move toward the upper edge, until, when the tilting is equal to or greater than four times the crown, all the surface water will drain to the low side. This must be considered in providing a sufficient number of properly located storm-water inlets.

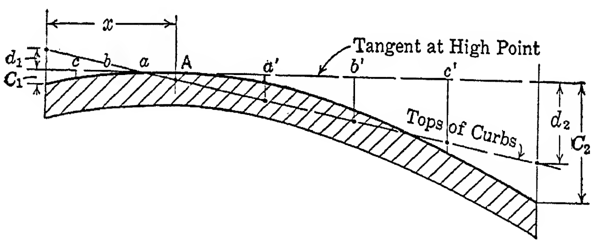


FIG. 56.

Occasionally it may be desired to make the high point of the crown near the upper edge, but provide some slope into the upper gutter. In Fig. 56, A is the high point of the crown at the distance x from the upper edge. C_1 is the amount of crown, measured from a tangent at the high point to the upper edge, and C_2 is the amount of

crown to the lower edge. The offsets a, b, and c are computed as usual, using x and C_1. The offsets a', b', and c' are not the same in this case, but are computed from the distance $W - x$ and the crown C_2. If d_1 is the distance from the tangent at the high point of the curve to the top of the curb or to the top of the grade stake adjacent to the upper edge of the pavement, and d_2 is the corresponding distance on the low side, the slope of the line of tops of curbs with respect to the tangent at the high point can be determined. The ordinates between these two lines at any points can then be found and the ordinates for use with a string or sight tees computed.

195. Subgrade Crowns. If the pavement is of uniform thickness, the subgrade crown has the same shape as the surface crown, and the ordinates to it from the top of curb are merely increased by the thickness of the pavement. If the pavement is not uniform in thickness, separate offsets for the subgrade must be computed to conform to its shape.

Horizontal Curves

The advantages of a parabola for a horizontal curve are: (1) it is not necessary to measure the intersection angle, (2) only the tape and simple sighting methods are needed to lay it out, (3) the calculations are extremely simple in the cases where the previous advantages apply, and (4) both equal and unequal tangents may be used. Theoretically it possesses slight easement properties as compared with the circle, but this is never the reason for using it. Its disadvantages are: (1) it is not easily laid out with the transit and tape by angle and line, (2) location with a tape is not as accurate as with a transit, (3) the entire area between tangents and long chord must be clear enough for field work, and (4) angles of intersection with lines crossing it are not easily measured or computed. It is used, therefore, only as a substitute for circular curves under conditions where its advantages outweigh its disadvantages. The experienced field man, however, finds many cases, especially on street work, where the parabola is exceedingly convenient.

There are two general methods of locating the parabola in the field with the tape, which may be termed the *offset method* and the *middle ordinate method*.

196. Offset Method. (*a*) *Equal tangents.* In Fig. 57, BA and AC are the two tangents, with the $P.I.$ at A, to be connected by the parabola having the tangent distance T. The tangent distance is

measured along both tangents and the $T.C.$ and the $C.T.$ are located. The long chord is then lined in, measured, and its middle point A' located. The line AA' is measured and its middle point a located. Point a is also the middle point of the curve, hence Aa is the external distance E, or the offset to the middle of the curve. The other offsets are computed as previously explained for vertical curves, since they vary as the square of the distances for chosen points on

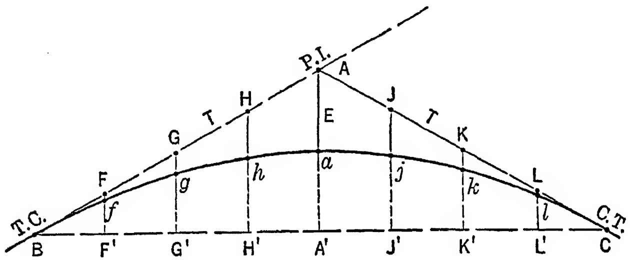


FIG. 57.

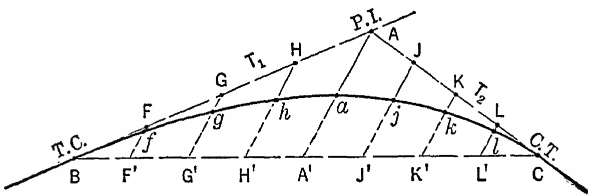


FIG. 58.

in Fig. 58, the parabola forms an excellent substitute for a compound curve. The curve is laid out in exactly the same way as for equal tangents, as explained in the preceding paragraph. In this case the dividing of the long chord to correspond with the tangents is of especial value in alining the offsets.

197. Middle Ordinate Method. In Fig. 59, the *T.C.* and *C.T.* and the middle point a of the curve are located as in the preceding method. $A'a$ is then the middle ordinate M. The chords Ba and aC are then lined in and their middle points G and K located. From these points perpendiculars to the chords, equal in length to $\frac{1}{4}Aa = \frac{1}{4}M$, are laid off which locate the points g and k. New chords

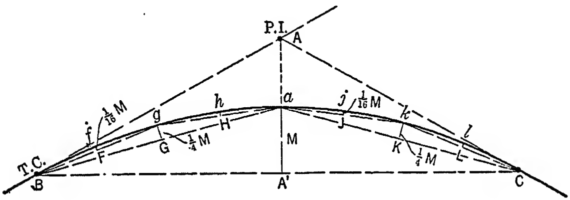


FIG. 59.

between these points may then be lined in and middle ordinates from them, equal to one-fourth the preceding middle ordinate, or $\frac{1}{16}M$, are laid off. This process of subdivision, making each new middle ordinate one-fourth of the one preceding, may be continued until enough points on the curve are located. This method can be used with either equal or unequal tangents.

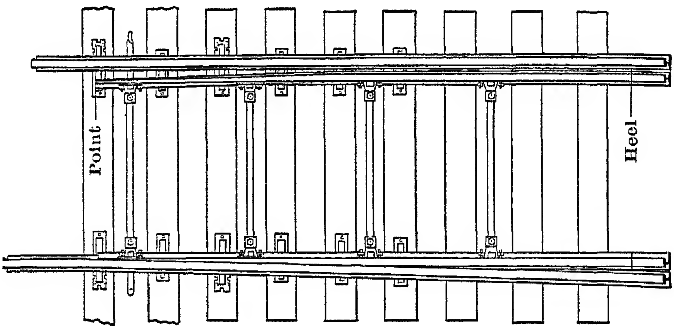
CHAPTER 8

RAILROAD TURNOUTS, CONNECTIONS, AND CROSSINGS

TURNOUTS

A TURNOUT is a combination of a switch, a frog, the rails necessary to connect the switch and the frog, two guard rails—unless the frog is self-guarded—and a switch-stand for operating the switch. A turnout begins with the switch and ends with the frog. The purpose of a turnout is to permit engines and cars to pass from one track to another.

198. Switches. A switch is a device to deflect at will the wheels of a train from the track upon which they are running. Switches are of three types, the split switch, the tongue switch, and the stub switch.



The *split switch* is the standard type used by railroads. It consists of two switch, or point, rails which are connected by tie-rods and operated as a unit, as illustrated in Fig. 60. The switch rails are of full section at one end, and are tapered to a $\frac{1}{4}$-in. point at the other end. The tapered end is called the *point of switch* and the other end is called the *heel of switch.* The switch rails rest upon

metal plates which are spiked to the ties. The heel of each switch rail is connected to its lead rail by means of the usual splice bars, and the switch as a unit pivots about these connections. The point of switch moves through a distance of about 5 in. which is called the *throw*. The movement of the switch rails is controlled by a switch-stand placed outside the track on a long tie, sometimes two, called a *head-block*. The distance between the gage lines of the main track and of the turnout at the heel of the switch rails is called the *heel spread* and varies from $5\frac{1}{2}$ to $6\frac{1}{4}$ in. The angle between the gage lines of the switch rail and of the main track rail is called the *switch angle*, *s*, and is computed from the equation,

$$\sin s = \frac{\text{heel spread } (h) - \text{width of switch point } (p)}{\text{length of switch rail } (l)}$$

Switch rails vary in length from 10 to 33 ft., depending on the curvature of the turnout. They are cut from rails of standard length so as to have no waste. For 30-ft. rails, 10, 12, 15, 18, 20, and 30-ft. switch points are used; and for 33-ft. rails, 11, 13, $16\frac{1}{2}$, 20, 22, and 33-ft. The increasing use of the 39-ft. rail will doubtless result in the use of the $19\frac{1}{2}$-ft. length of switch rail. The American Railway Engineering Association recommends the use of the 11-ft. switch rail with the No. 6 frog, the 16.5-ft. switch rail with Nos. 7, 8, and 10 frogs, the 22-ft. switch rail with the No. 11 frog, and the 30-ft. switch rail with Nos. 16 and 20 frogs.

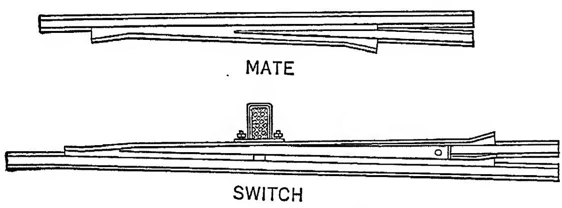


FIG. 61.—Tongue Switch.

The *tongue switch*, Fig. 61, consists of a steel wedge or tongue pivoted at one end and moving in a heavy cast-steel frame so arranged that pavement may be built around it. This is the common form on street railways, where it is generally used singly with a rigid fitting called a *mate* in the opposite rail. Steam roads use them only for turnouts in pavement, and then usually in pairs with

the tongues connected by a tie-rod, where the action is the same as in a split switch.

The *stub switch*, Fig. 62, consists of a pair of ordinary rails fastened together with tie-rods. The rails are spiked to the ties for part of their length, or else are short and are held only by the splice bars at the fixed end, the free end being thrown to match the stub ends of the lead rails. A stub switch differs from a split switch in that the toe is fixed and the heel is moved. This form of switch may be employed to advantage on construction and mine tracks,

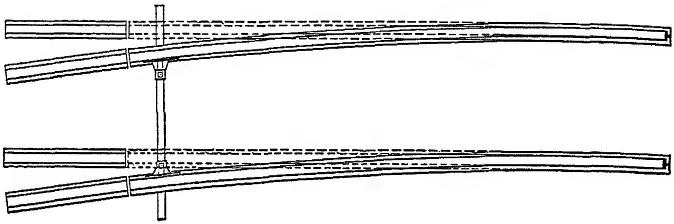


FIG. 62.—Stub Switch.

owing to its cheapness and ease of installation. *A stub switch should never be used on a standard road.*

199. Frogs. A frog is a device at the intersection of two running rails to permit the flanges of wheels moving along one rail to cross the other rail. Turnout frogs may be classified as (1) rigid frogs, and (2) spring-rail frogs. Both types of frogs are made with straight gage lines, except those used on street railways. The main point rail is finished with a blunt point about $\frac{1}{2}$ in. wide. The distance P between this *actual* frog point and the *theoretical* point, or intersection of gage lines, equals the width of the blunt point multiplied by the frog number.

Fig. 63 shows the common form of rigid frog and the names of its principal parts. As its name implies, the rigid frog has no movable

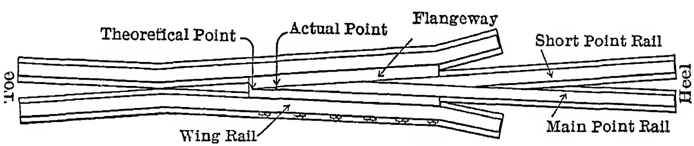


FIG. 63.—Rigid Frog.

parts. The distinguishing feature of the rigid frog is that both flangeways are open and must be jumped by passing wheel treads. This type of frog is used chiefly in turnouts from secondary tracks.

Fig. 64 shows the common form of a spring-rail frog. In this type one wing rail, and sometimes both, is made movable. The

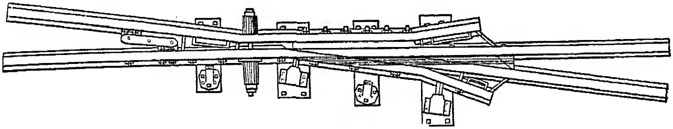


FIG. 64.—Spring-rail Frog.

wing rail is normally held against the point rail by springs. A wheel moving along the main track, therefore, has an open flangeway and a continuous support for the tread, while a wheel moving along the turnout track must spring the wing rail open to allow the

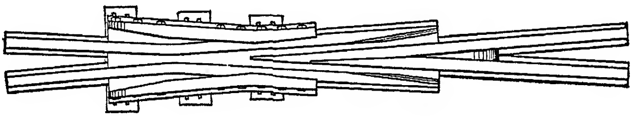


FIG. 65.—Self-guarded Frog.

flange to pass, and must jump the open flangeway as in the case of the rigid frog. Guard rails are essential to the safe operation of a spring-rail frog. This type of frog is generally used with turnouts from main-line track.

200. Guard Rails. Guard rails are short pieces of rails, with flared ends, spiked, bolted, or clamped against the outside rails of both main and turnout tracks opposite the frog, so as to leave a flangeway of $1\frac{3}{4}$ in. The function of the guard rail is to force the flange of a wheel on the outside rail against the head of that rail and thus to prevent the flange on the inside wheel from striking the point of the frog. Guard rails in place are illustrated in Fig. 66.

Self-guarded frogs are used extensively. This type of frog eliminates the separate guard rails, as it has, as an integral part of its structure, raised guards on the wing rails which bear against the face of the wheel and thus guide the wheel flanges past the frog point. A self-guarded frog is shown in Fig. 65.

201. Frog Angle, *F*. The frog angle is the angle between the gage lines of the frog at their point of intersection. Obviously, the frog angle may have any value, but only values between 2° and 8° are practical for railroad turnouts. Larger values may be used on mine, industrial, and street-railway tracks. A few standard values which meet all practical requirements have been chosen by the railroads to

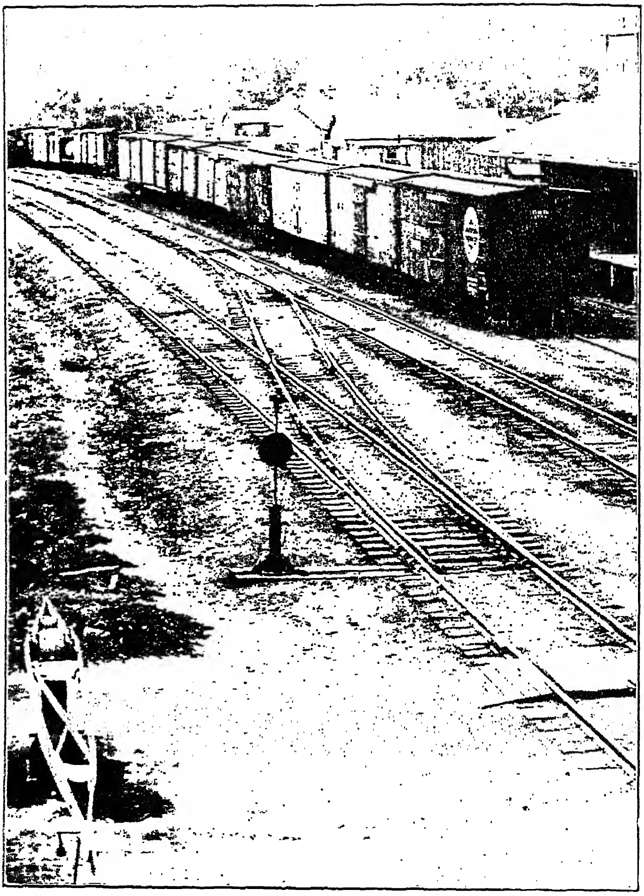


FIG. 66.—Crossover.

agree with simple values of the frog number, which is a more convenient method of designating frogs. The frog number is defined in the following section. The values of standard frog angles used are given in column 2, Table 6.

202. Frog Number, N. The frog number is the ratio of the axial length to the spread, that is, it is the distance, measured along the bisector of the frog angle, in which the gage lines diverge a unit distance. Expressed as a trigonometric function,

$$N = \tfrac{1}{2} \cot \tfrac{1}{2}F \qquad (67)$$

Values of N from 7 to 16 are commonly used, but for special purposes values from 3 to 27 are occasionally used. The American Railway Engineering Association recommends the use of Nos. 16 and 20 frogs for main-line, high-speed movements, Nos. 10 and 12 for main-line, low-speed movements, and No. 8 for yards and sidings.

203. Location of Turnouts. The first step in staking out a turnout is to establish the position of the frog. The heel or the toe of the frog should come at a regular rail joint in order to avoid short pieces of rail in the track. Knowing the length of the frog to be used, the position of the point of frog, *P.F.*, can be fixed. The next step is to mark the position of the point of switch, *P.S.* This is done by measuring along the main track a distance which is called the *lead*, L. If this brings the point of switch too near a rail joint, the position of the frog must be changed, making the toe of the frog come at a rail joint instead of the heel, or vice versa. Having satisfactorily established the positions of the point of frog and the point of switch, the outside rail of the turnout curve is then located. This is done by measuring offsets from the gage line of the outside main rail at the middle and quarter points of the turnout curve. The inner turnout rail is set by gage from the outer rail.

204. Turnouts from Straight Track. Since the frog and the switch rails are straight, the alinement of a turnout is not a simple curve throughout, but consists of two short pieces of tangent connected by a simple curve.

The solution of the turnout curve is as follows:

In triangle 1, Fig. 67, the hypotenuse is the distance from toe to point of frog, and the angles are known. Solve the triangle for the base m and the altitude n.

In triangle 2, the altitude, $q = g - h - n$, and the angles are known. Solve the triangle for the base j and the hypotenuse c. c is also the long chord of the outside rail of the turnout curve, whose central

angle is $F - s$; hence, the radius of the turnout curve can be computed from the equation,

$$R + \tfrac{1}{2}g = \frac{c}{2 \sin \frac{1}{2}(F - s)} \tag{68}$$

The distance L along the straight main track from the $P.S.$ to the $P.F.$ is

$$L = l + j + m + P \tag{69}$$

To compute the offsets from the main track rail to the outside rail of the turnout curve, extend the turnout curve through the switch angle s until its tangent is parallel to the main rail. The distance between these tangents is

$$e = h - (R + \tfrac{1}{2}g) \text{ vers } s \tag{70}$$

The offset from the gage line of the main track rail to the gage line of the turnout curve at its quarter point is then,

$$y_1 = e + (R + \tfrac{1}{2}g) \text{ vers } [s + \tfrac{1}{4}(F - s)] \quad \text{(Slide Rule)}$$

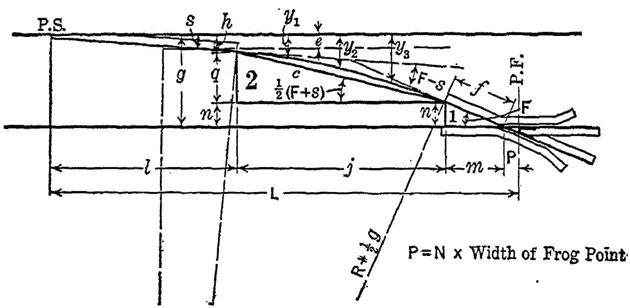


FIG. 67.

For the middle point,

$$y_2 = e + (R + \tfrac{1}{2}g) \text{ vers } [s + \tfrac{1}{2}(F - s)] \quad \text{(Slide Rule)}$$

And for the three-quarters point,

$$y_3 = e + (R + \tfrac{1}{2}g) \text{ vers } [s + \tfrac{3}{4}(F - s)] \quad \text{(Slide Rule)}$$

205. Turnouts from Curved Track. It can be shown—the solution is too involved to be included here—that the lead of a turnout from a curved track is practically the same as the lead of a turnout from a straight track with the same frog number. Also, the degree of the turnout curve is equal to the degree of curve of the turnout from a straight track increased or diminished by the degree of the main track curve, depending on whether the turnout is on the inside or on the outside of the main track curve. A turnout from a curved track, therefore, is staked out with the same dimensions as one with the same frog number from a straight track.

206. Double Turnouts. Occasionally two turnouts to opposite sides of the main track are located at the same point. This involves the use of two regular frogs in the main rails, a third or *crotch frog* at the intersection of the lead rails, and a *three-throw switch.* Such a switch is structurally weak and should never be used in a main track. And further, the conditions under which such arrangements are absolutely necessary are extremely rare. It is better practice to use two separate turnouts, placing one switch ahead of the other and, if necessary, using a sharper turnout to save distance, than to use a three-throw switch. Double turnouts will therefore not be further considered.

207. Practical Leads. Since it is undesirable to use short pieces or odd lengths of rails, it is the universal practice to modify the theoretical leads as computed in Eq. 69 so as to use more convenient lengths of lead rails. The difference between the theoretical lead and the practical lead is never very great, and the turnout is located as previously explained except for this modification in the lead. Changing the lead has the effect of changing the degree of the turnout curve.

Every railroad has its standard turnouts, all dimensions of which are worked out and tabulated. The turnouts are then installed by the trackmen, the only duty required of the engineer being to locate the *P.F.* and occasionally the *P.S.* The engineer, therefore, has little occasion to use the functions of the turnout proper. His problem is to connect the turnout to the required track.

Table 6 gives the practical turnout leads and length of switch rail recommended by the American Railway Engineering Association; also, the rectangular coordinates to the quarter points on the gage side of the outside turnout rail, referred to the point of switch as origin.

TABLE 6

A.R.E.A. Standard Straight Switch Turnouts

Frog				Switch		Turnout											
Frog number	Frog angle	Length, feet		Length, feet	Switch angle	Lead, actual *P.S.* to actual *P.F.*, feet	Center-line curve		Coordinates from point of switch for quarter and center points to gage line of curved closure rail, feet						Closure rails	
		Total	Point to heel				Radius, feet	Degree	x_1	x_2	x_3	y_1	y_2	y_3	Straight, feet	Curved, feet
5	11° 25′ 16″	9.00	5.46	11.00	2° 39′ 34″	42.54	177.80	32° 39′ 56″	18.00	25.00	32.00	0.984	1.719	2.740	28.00	28.33
6	9° 31′ 38″	10.00	6.25	11.00	2° 39′ 34″	47.50	258.57	22° 17′ 58″	19.19	27.37	35.56	1.031	1.800	2.833	32.75	33.00
7	8° 10′ 16″	12.00	7.29	16.50	1° 46′ 22″	62.08	365.59	15° 43′ 16″	26.19	35.85	45.56	0.948	1.630	2.573	40.96	41.10
8	7° 09′ 10″	13.00	7.92	16.50	1° 46′ 22″	68.00	487.28	11° 46′ 44″	27.60	38.71	49.81	0.980	1.719	2.694	46.42	46.62
9	6° 21′ 35″	16.00	8.62	16.50	1° 46′ 22″	72.29	615.12	9° 19′ 30″	28.85	41.21	53.56	1.036	1.781	2.786	49.50	49.60
10	5° 43′ 29″	16.50	10.08	16.50	1° 46′ 22″	78.75	779.39	7° 21′ 24″	29.98	43.46	56.94	1.021	1.750	2.719	55.83	56.00
11	5° 12′ 18″	18.71	11.71	22.00	1° 19′ 46″	91.85	927.27	6° 10′ 56″	37.71	53.42	69.12	1.021	1.719	2.813	62.85	63.00
12	4° 46′ 19″	20.33	12.54	22.00	1° 19′ 46″	96.67	1104.63	5° 11′ 20″	38.71	55.42	72.12	1.036	1.802	2.823	66.87	67.00
14	4° 05′ 27″	23.58	14.96	22.00	1° 19′ 46″	107.06	1581.20	3° 37′ 28″	41.10	60.21	79.31	1.073	1.859	2.875	76.44	76.56
15	3° 49′ 06″	24.37	14.96	30.00	0° 58′ 30″	126.38	1720.77	3° 19′ 48″	51.75	73.50	95.25	1.010	1.771	2.813	86.96	87.06
16	3° 34′ 47″	26.00	16.58	30.00	0° 58′ 30″	131.33	2007.12	2° 51′ 18″	53.00	76.00	99.00	1.036	1.818	2.859	91.92	92.00
18	3° 10′ 56″	29.25	18.21	30.00	0° 58′ 30″	140.96	2578.79	2° 13′ 20″	55.0	80.00	105.00	1.063	1.844	2.870	99.92	100.00
20	2° 51′ 51″	30.87	19.83	30.00	0° 58′ 30″	151.96	3289.29	1° 44′ 32″	57.75	85.50	113.25	1.088	1.891	2.932	110.92	111.00

Frog Point $\frac{1}{2}''$ wide. Switch Point $\frac{1}{8}''$ wide. Switch Heel Spread $6\frac{1}{4}''$.

CONNECTIONS

Connections are the tracks used to connect a turnout from a main track with (1) a diverging track, (2) a parallel siding, or (3) a turnout on another track. These three classifications are treated separately in the following sections.

Diverging Tracks

208. Case 1. From Straight Track.

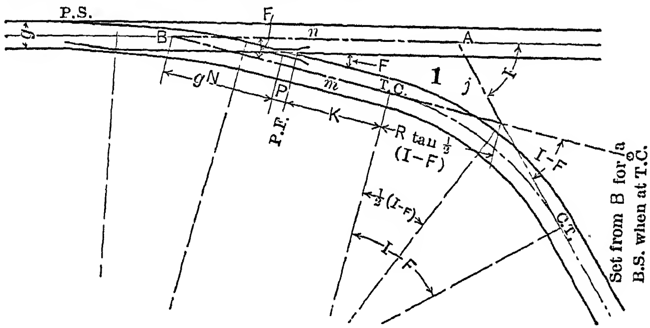


FIG. 68.

The solution of this problem requires that I, N, R, K, and P be known. I is the intersection angle between the diverging track and the straight main track. It is measured in the field after the point of intersection, A, has been determined. The angle should be measured to the nearest one-half minute.

N is the frog number chosen for the turnout. The value of N is determined by the standard practice of the individual railroad.

R is the radius chosen for the connecting curve. The factors which affect the choice of R and D are (1) the degree of the turnout curve, (2) the topography, and (3) special space limitations. Of these factors, the first is generally the one which determines the value of R. In general, an even value of D is chosen which is slightly less than the degree of the turnout curve. The degrees of the turnout curves for the various frog numbers are given in column 9, Table 6.

K is the frog tangent, or the length of tangent between the actual point of frog and the $T.C.$ of the connecting curve. The minimum

value of K is the distance from the actual point to the heel of the frog. Generally, values of K from 20 to 40 ft. are chosen, the larger values being adapted to the sharper curves.

P is the distance from the actual to the theoretical point of frog. It is equal to the width of the blunt point of the frog multiplied by the frog number. This width is usually $\frac{1}{2}$ in.

The solution of this problem requires the computation of the data necessary to locate (1) the $P.F.$, (2) the $T.C.$, and (3) the $C.T.$

In triangle 1, Fig. 68, all the angles and the side, $m = gN + P +$

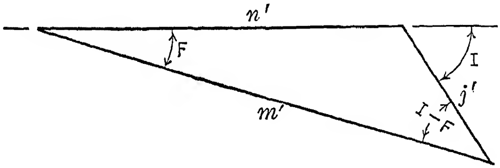


FIG. 69.

$K + R \tan \frac{1}{2}(I - F)$, are known, whence the sides n and j can be computed.

The point B and the $C.T.$ of the connecting curve are located from point A by the distances n and $j + R \tan \frac{1}{2}(I - F)$, respectively. The $P.F.$ is located from B by the coordinates $gN + P$ and $\frac{1}{2}g$. The $T.C.$ of the connecting curve is located from B by the angle F and the distance $gN + P + K$. For standard gage, $g = 4.708$ ft.

The frog, as located in the preceding paragraph, will rarely fit the rail joints of an existing track and will have to be moved either forward or backward until one end does meet a rail joint. This involves a change in the location of the point B and the $C.T.$, and a change in the length of the frog tangent K. The amount of movement is found in the field by locating the $P.F.$ with the computed dimensions and then with the known dimensions of the frog, measuring the distance from either end of the frog to the nearest rail joint.

Obviously triangle 1 could be recomputed using the revised value of the side n, but it is simpler to determine the *corrections* to all dimensions by using a correction triangle, which can always be solved with a slide rule and often mentally with sufficient accuracy.

In Fig. 69, n' is the distance the frog must be moved, j' is the amount the $C.T.$ must be shifted, and m' is the change of length

of the frog tangent K. All of these will be negative if the frog is moved forward and positive if it is moved backward.

This triangle may be solved trigonometrically, but it is often easier to solve it by simple proportion since:

$$j' : j :: n' : n \quad \text{and} \quad m' : m :: n' : n$$

The corrections, j', m', and n', are then applied to the computed dimensions, j, m, and n, with the proper sign and the points located in the field as previously outlined. The connecting curve is then run in. It is good practice to back it in from the $C.T.$, since a longer backsight for orienting the transit can be obtained at this point than at the $T.C.$ and it saves a set-up.

209. Case 2. Turnout from the Inside of a Curved Main Track.

(*a*) *Points A and H on circular curve.*

The data and the requirements of this problem are the same as in the preceding problem, except that the main track is on a $D°$ curve.

In triangle 1, Fig. 70, the hypotenuse and the angles are known, and the other two sides are computed as shown in the figure.

In triangle 2, the base and the altitude can be found by arithmetic. Solve the triangle for the hypotenuse and the angle a.

In triangle 3, the hypotenuse and the angles are known, and the other two sides are computed as shown in the figure.

In triangle 4, the hypotenuse OO_1 is known from triangle 2, and the side O_1J can be found by arithmetic. Solve the triangle for the side OJ and the angle b.

Then

$$x = a - b + (90 - I) - (90 - F)$$

$$= a - b - I + F \tag{71}$$

and

$$y = a - b \tag{72}$$

The point H opposite the theoretical point of frog is located from point A by measuring along the center line the distance AH (in feet) $\frac{x}{D}$ 100. The $C.T.$ is located from point A by the distance AE $AG - EG = AG - JO$. The $T.C.$ is best located as follows: Set-up over H; back-sight on A; turn off $\frac{1}{2}x$ to get on tangent; locate M for a temporary back-sight; plunge telescope and set point B a dis-

tance gN from H; set-up at B and back-sight on M; turn off the frog angle and locate the $T.C.$ on this line a distance $gN + P + K$ from B; set point Q for a temporary back-sight; set-up at the $T.C.$, back-sight on Q, and run in the connecting curve to point E.

It is to be especially noted that, when $R \cos I$ is less than R_1, the line OJ, Fig. 70, falls on the other side of the line OO_1, and the *sign of the angle b in Eqs.* 71 *and* 72 *becomes plus.*

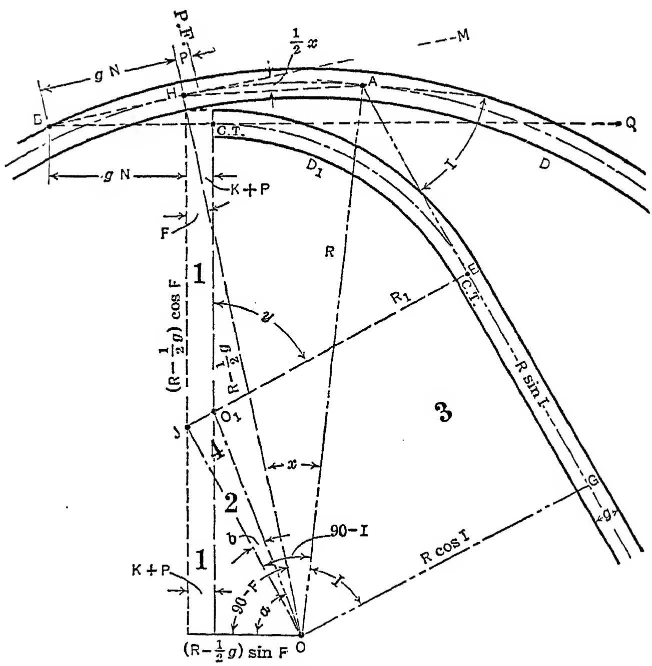


FIG. 70.

The frog as located opposite H in Fig. 70 will rarely come at an existing rail joint in the main track, and it must be moved forward or backward until it does. This involves a change in the location of the point B and of the $C.T.$ and a change in the length of the frog tangent K. The solution is as follows: In Fig. 71, the points A, H, B, $T.C.$, and $C.T.$ have the same positions as in Fig. 70. It

is necessary to move the *P.F.* forward, as shown, a measured distance d to a point opposite H'. The angle z through which the frog is moved is found from the equation,

$$z \text{ (in minutes)} = 0.6dD \qquad (73)$$

The moving of the *P.F.* makes the following changes in the track layout:

1. The direction of the frog tangent is changed an amount equal to the angle z.
2. The length and the position of the frog tangent are changed.
3. The central angle y of the connecting curve is decreased by the angle z, thereby decreasing the length of the connecting curve.
4. The entire connecting curve is moved parallel to the tangent AE.

It is required to find the length of the new frog tangent and the distance the *C.T.* is shifted.

In triangle 1, Fig. 71, the angles and the hypotenuse are known.

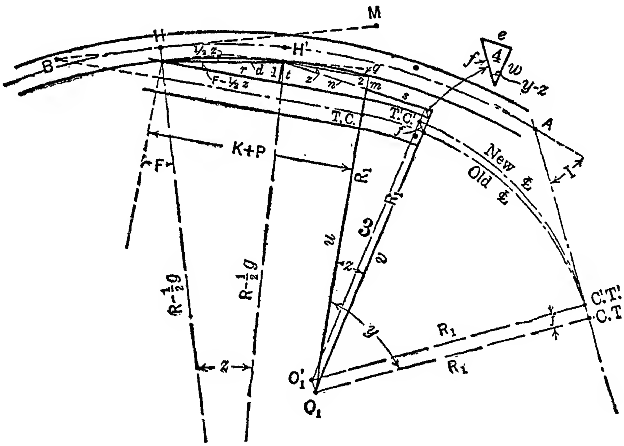


FIG. 71.

Solve the triangle for the base r and the altitude t. This triangle and those that follow may be solved on the slide rule.

In triangle 2, the base, $q = K + P - r$, and the angles are known. Solve the triangle for altitude m and the hypotenuse n:

In triangle 3, the hypotenuse, $u = R_1 + \frac{1}{2}g + t - m$, and the angles are known. Solve the triangle for the base v and the altitude s.

In triangle 4, the base, $w = v - R_1 - \frac{1}{2}g$, and the angles are known. Solve the triangle for the altitude e and the hypotenuse f. Then

$$\text{The new} \qquad K + P = n + s - e$$

And

$$\text{The change in the } C.T. = f$$

If the frog is moved backward, another figure is required for an accurate solution. Although this figure will have a somewhat different appearance, the four triangles necessary for the solution are constructed in the same manner as those shown in Fig. 71. For example, triangle 1 is constructed with a hypotenuse equal to d and with a base and altitude formed by dropping a perpendicular from the old $P.F.$ to a line through the new frog point parallel to the old frog tangent. The angle at H', however, now becomes $F + \frac{1}{2}z$ instead of $F - \frac{1}{2}z$. The student should construct this figure as an exercise. The change in the length of the frog tangent and the change in the position of the $C.T.$, however, are approximately the same as when the frog is moved forward, but with opposite sign. Compare the answers to problems 5 and 6, page 227.

The solution of this problem, as given in Figs. 70 and 71, is based on the assumption that the alinement of the main track is perfect. In many instances, however, the main line curve will be found to be more or less out of correct alinement. In these cases, the problem is best solved as follows:

1. Solve the triangles in Fig. 70 approximately only, using a slide rule or three-place tables. Measure from A the computed distance AH, and locate point H.

2. Locate the point of frog so that the heel or the toe of the frog will come at the rail joint nearest to the point H.

3. Set-up the transit in the center of the track opposite the selected point of frog, and establish the tangent to the curve at this point by sighting at a point in the center of the track 100 ft. ahead of the instrument, plunging the telescope, measuring the deflection angle to a point in the center of the track 100 ft. back of the instrument, and bisecting this deflection angle.

4. Measure back along the tangent thus established the dis-